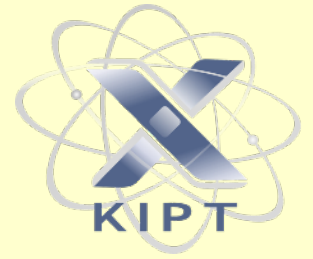


National Academy of Sciences of Ukraine
National Science Center
“Kharkiv Institute of Physics and Technology”
V.N. Karazin Kharkiv National University



Lecture #3: Nuclear Burning Wave Phenomenon & Traveling Wave Reactor

Sergii P. Fomin

*Leading researcher, PhD, Akhiezer Institute for Theoretical Physics
National Science Center “Kharkiv Institute of Physics and Technology”*

e-mail: spfomin@gmail.com

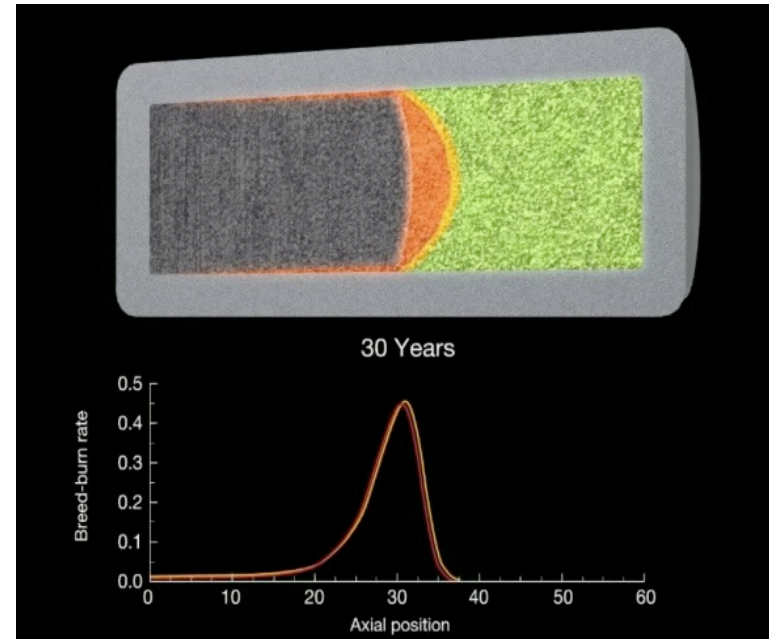
Outlook:

- Nuclear Burning Wave phenomenon
- Mathematical approach & calculation results
- Mixed Th-U-Pu fuel cycle
- Stability study of NBW regime
- Negative reactivity feedback & intrinsic safety
- Transient processes in NBW reactor
- Main features of NBW reactor & unsolved problems

“A New Nuclear Evangelist”

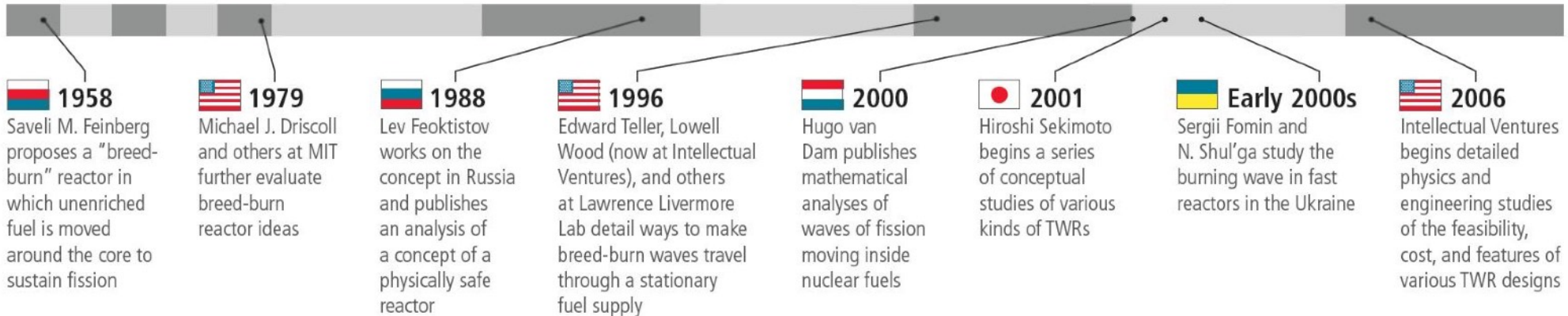


http://www.ted.com/talks/bill_gates.html

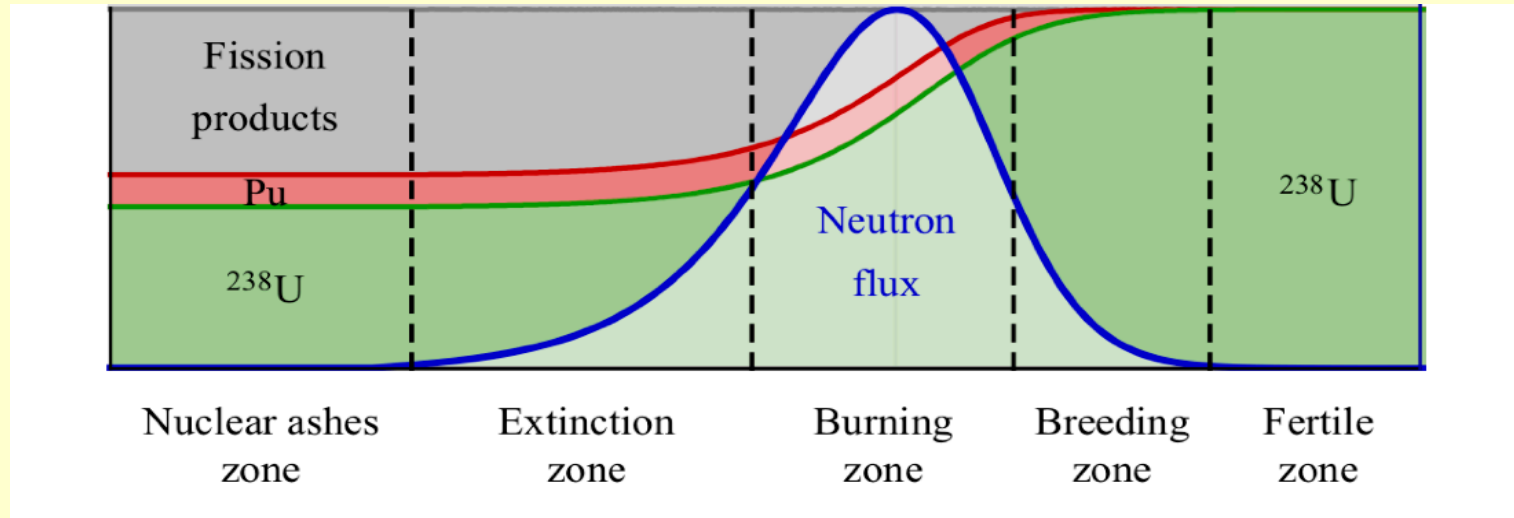


TerraPower + Toshiba + China + Korea Rep. = TWR (2020)

The Evolution of the Traveling-Wave Concept

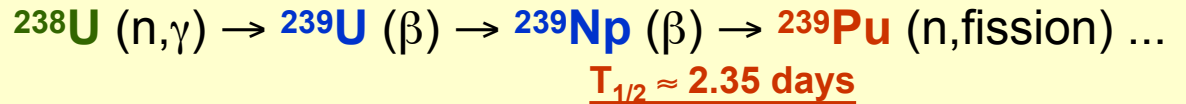


Lev Feoktistov (USSR, 1988): Nuclear Burning Wave concept



L.P. Feoktistov. Preprint IAE-4605/4, 1988.

L.P. Feoktistov. *Sov. Phys. Doklady*, 34 (1989) 1071.



$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial z^2} + vn \left(\sigma_{a8} N_8 - (\sigma_a + \sigma_f)_{\text{Pu}} N_{\text{Pu}} \right)$$

$$\frac{\partial N_8}{\partial t} = -vn\sigma_{a8} N_8 ; \quad \frac{\partial N_9}{\partial t} = vn\sigma_{a8} N_8 - \frac{1}{\tau_\beta} N_9$$

$$\frac{\partial N_{\text{Pu}}}{\partial t} = \frac{1}{\tau_\beta} N_9 - vn (\sigma_a + \sigma_f)_{\text{Pu}} N_{\text{Pu}}$$

$$N_{cr}^{\text{Pu}} = \frac{\sum_i \sigma_{ai} N_i}{(v-1)\sigma_f^{\text{Pu}}}$$

$$N_{eq}^{\text{Pu}} = \frac{\sigma_{a8} N_8}{\sigma_f^{\text{Pu}} + \sigma_a^{\text{Pu}}}$$

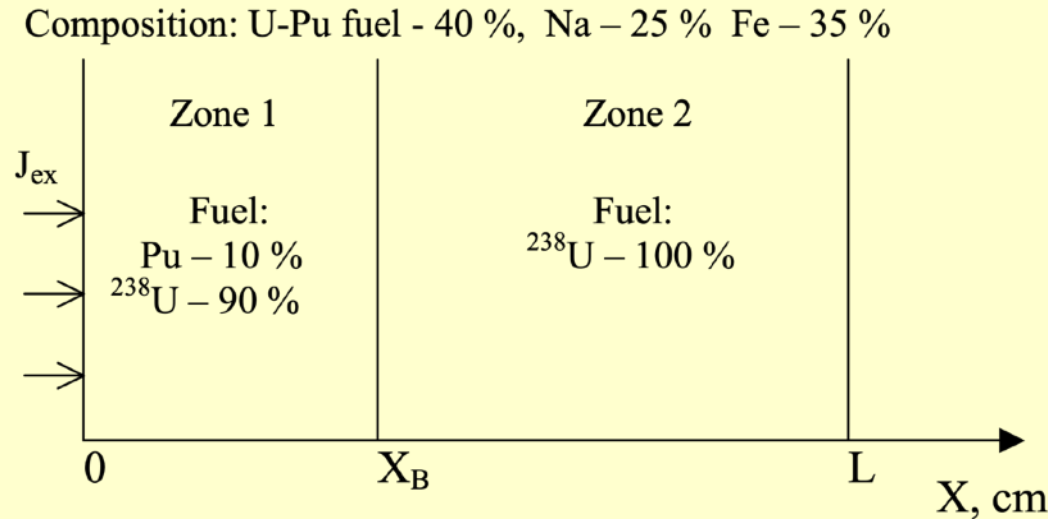
$x = z + Vt$

$N_{eq}^{\text{Pu}} > N_{cr}^{\text{Pu}}$

Feoktistov criterion

1992 V. Goldin, D. Anistratov (Moscow Institute of Applied Mathematics)

V. Goldin, D. Anistratov, Preprint IAM, # 43, 1992; Mathematical Modelling, 7 (1995) 12.



Non-stationary problem !
1d one-group approximation
(U-Pu fuel cycle)

$$\frac{\partial C_l^i}{\partial t} = -\lambda_l^i C_l^i + \beta_l^i (v_f \Sigma_f)_l \Phi$$

$$C_l^i(x, t = 0) = C_{0l}^i(x)$$

$$\frac{1}{v} \frac{\partial \Phi}{\partial t} - \frac{\partial}{\partial x} \left(D \frac{\partial \Phi}{\partial x} \right) + \Sigma_a \Phi - (1 - \bar{\beta}) (v_f \Sigma_f) \Phi = \sum_l \sum_i \lambda_l^i C_l^i$$

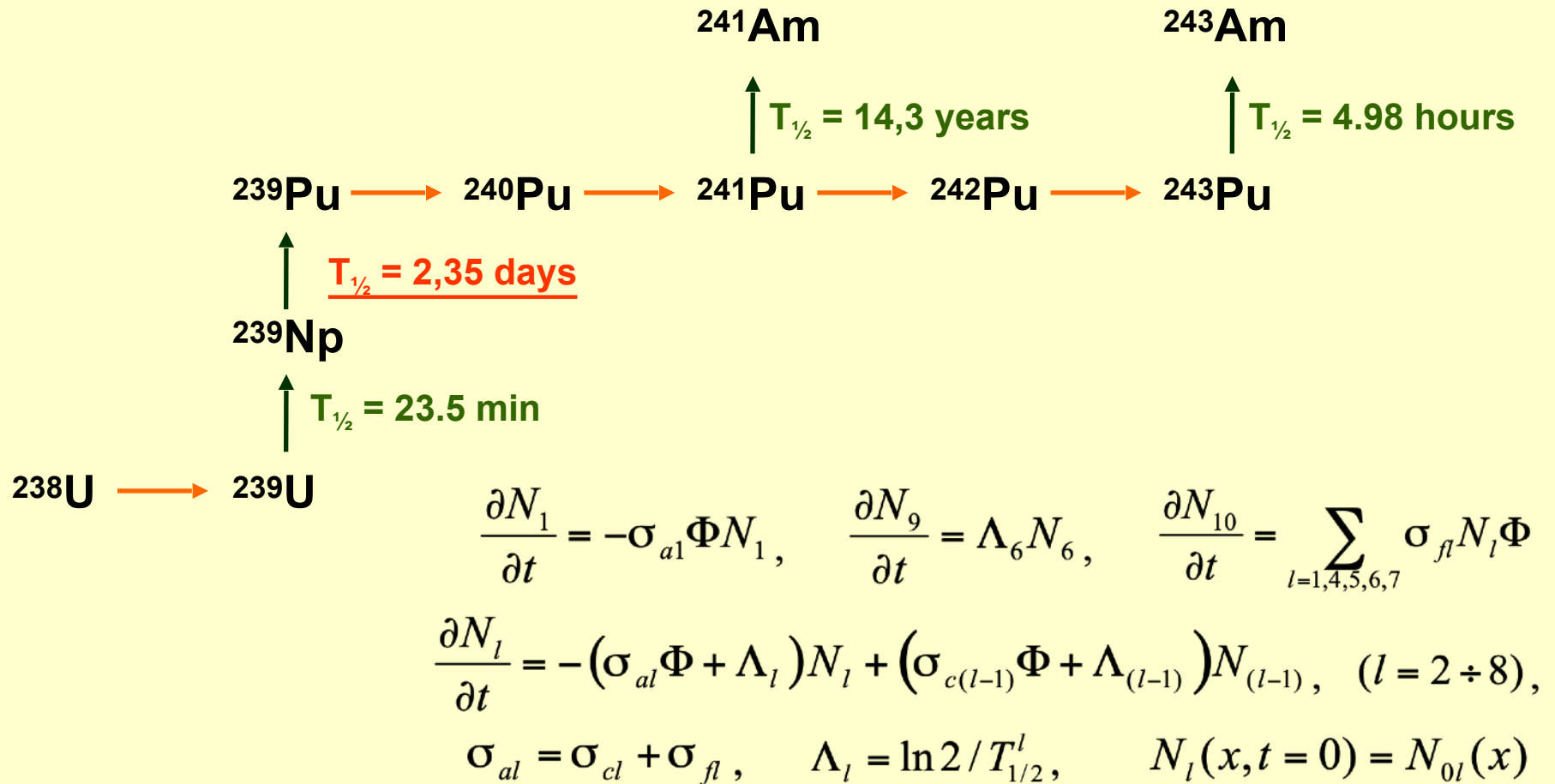
$$\bar{\beta} = \sum_l \beta_l (v_f \Sigma_f)_l / v_f \Sigma_f$$

$$D(x) = 1/3 \Sigma_{tr}(x) \quad \Sigma_\alpha(x) = \sum_j \sigma_\alpha^j N^j(x) \quad v_f \Sigma_f = \sum_j v_f^j \sigma_f^j N^j(x) \quad \beta_l = \sum_i \beta_l^i$$

$$\Phi(0) - 2D(0) \frac{\partial \Phi(x)}{\partial x} \Big|_{x=0} = 2j_{ex} \quad \Phi(L) + 2D(L) \frac{\partial \Phi(x)}{\partial x} \Big|_{x=L} = 0 \quad D'(x) \frac{d\Phi'(x)}{dx} = D''(x) \frac{d\Phi''(x)}{dx}$$

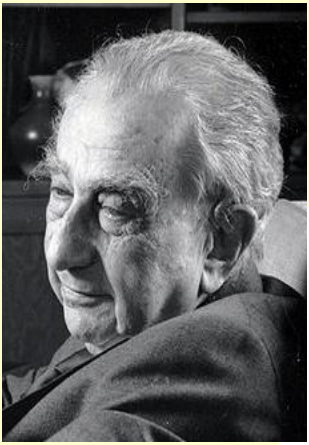
$$\Phi'(x) = \Phi''(x) \quad \Phi(x, t = 0) = 0 \quad 0 \leq x \leq L \quad 0 \leq t \leq T$$

Dynamics of the FR nuclear composition



The numeration of the nuclei in the U–Pu transformation chain

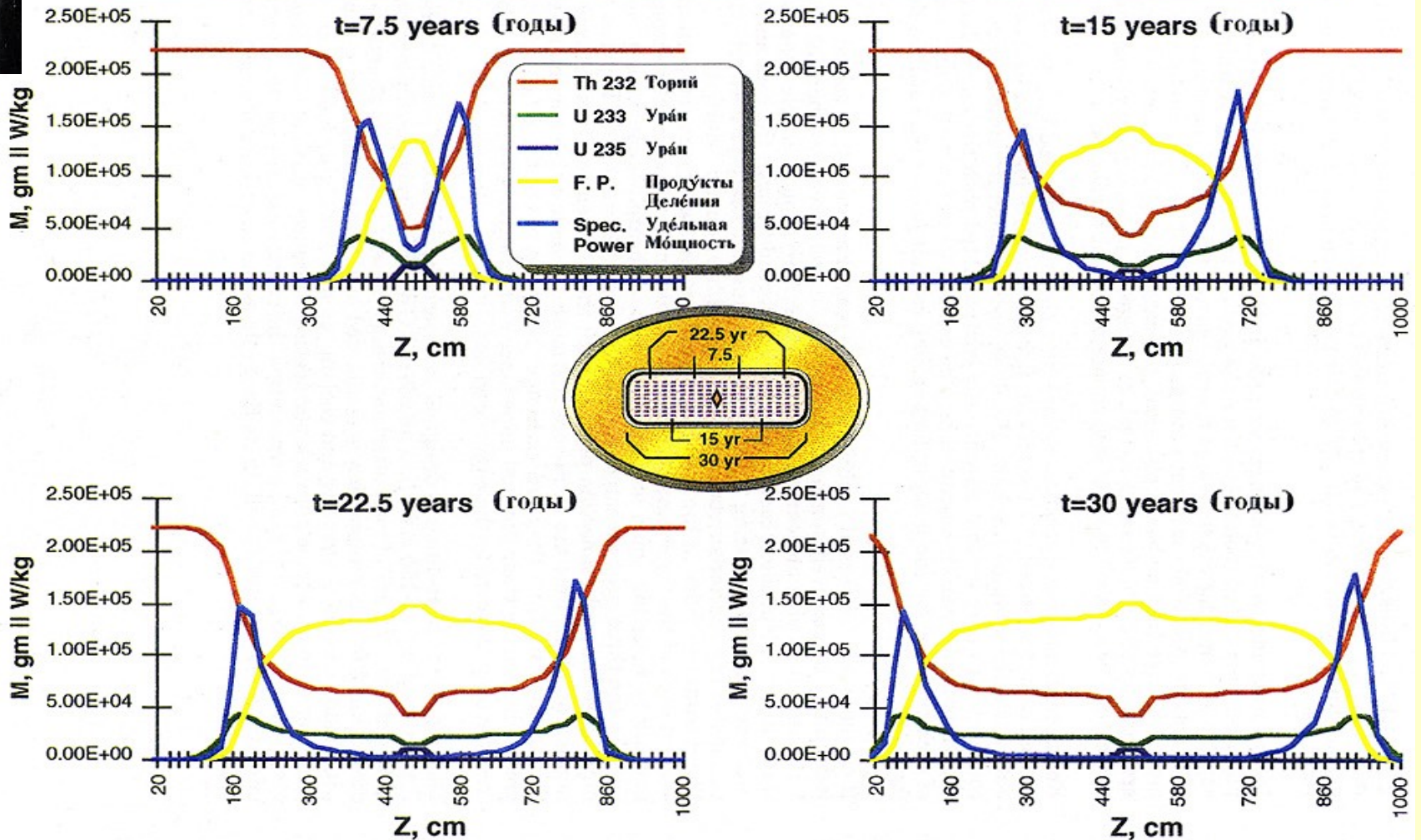
N	1	2	3	4	5	6	7	8	9	10
Nucleus	^{238}U	^{239}U	^{239}Np	^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu	^{243}Am	^{241}Am	FP



Edward Teller: Nuclear Energy for the Third Millennium

Preprint UCRL-JC-129547, LLNL, 1997. - Th-U fuel cycle

Salient Features Of Nuclear Deflagration Wave Propagation (Full-Power Case)

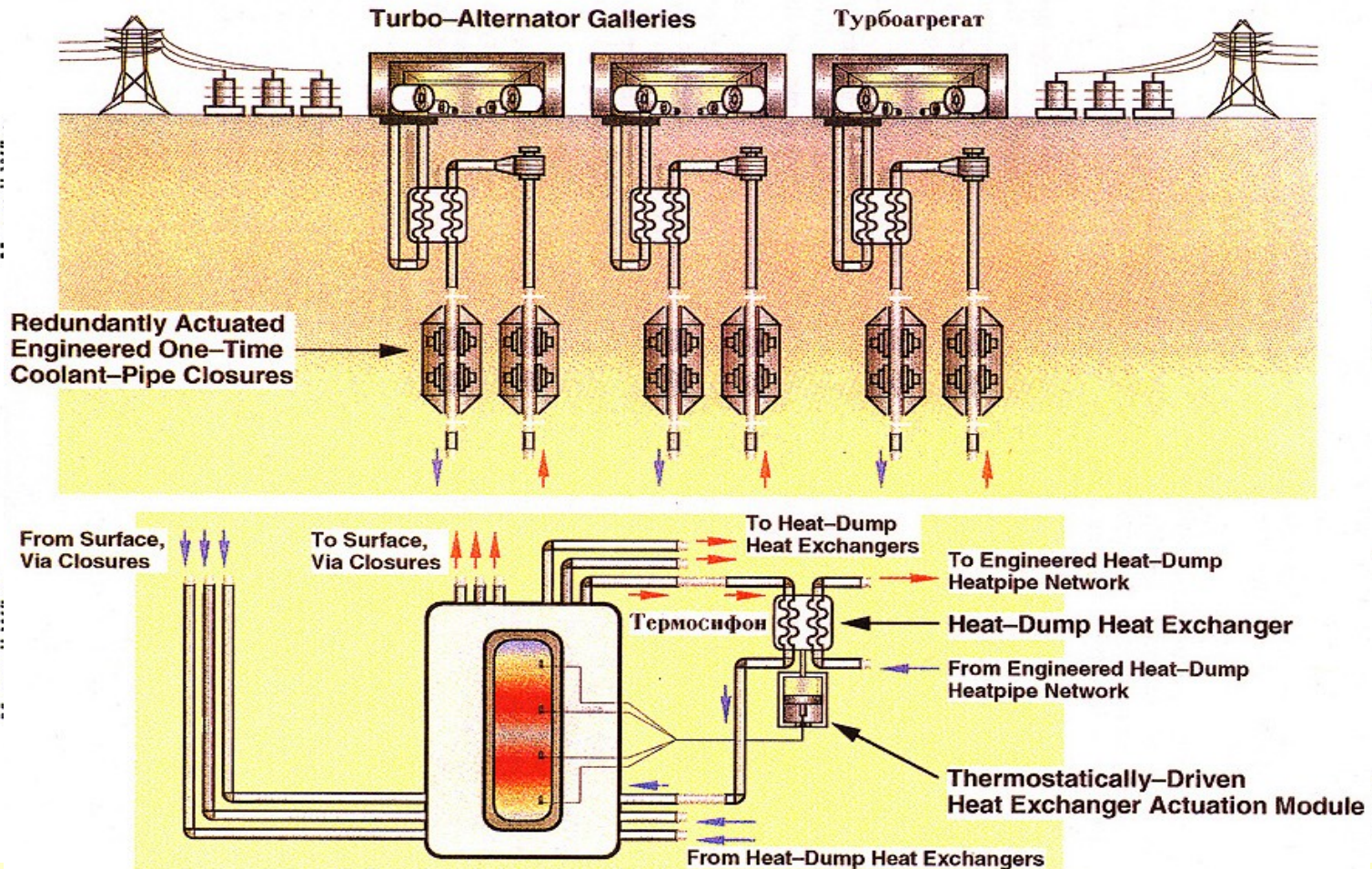


Edward Teller (LLNL, USA) 1997: Traveling Wave Reactor

E.Teller, 1997. *Nuclear Energy for the Third Millennium*. Preprint UCRL-JC-129547, LLNL.

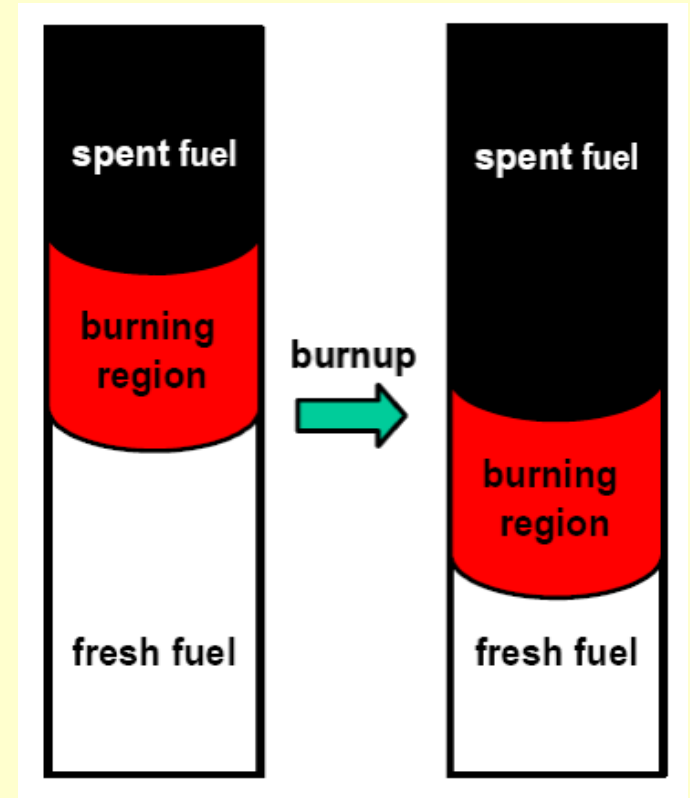
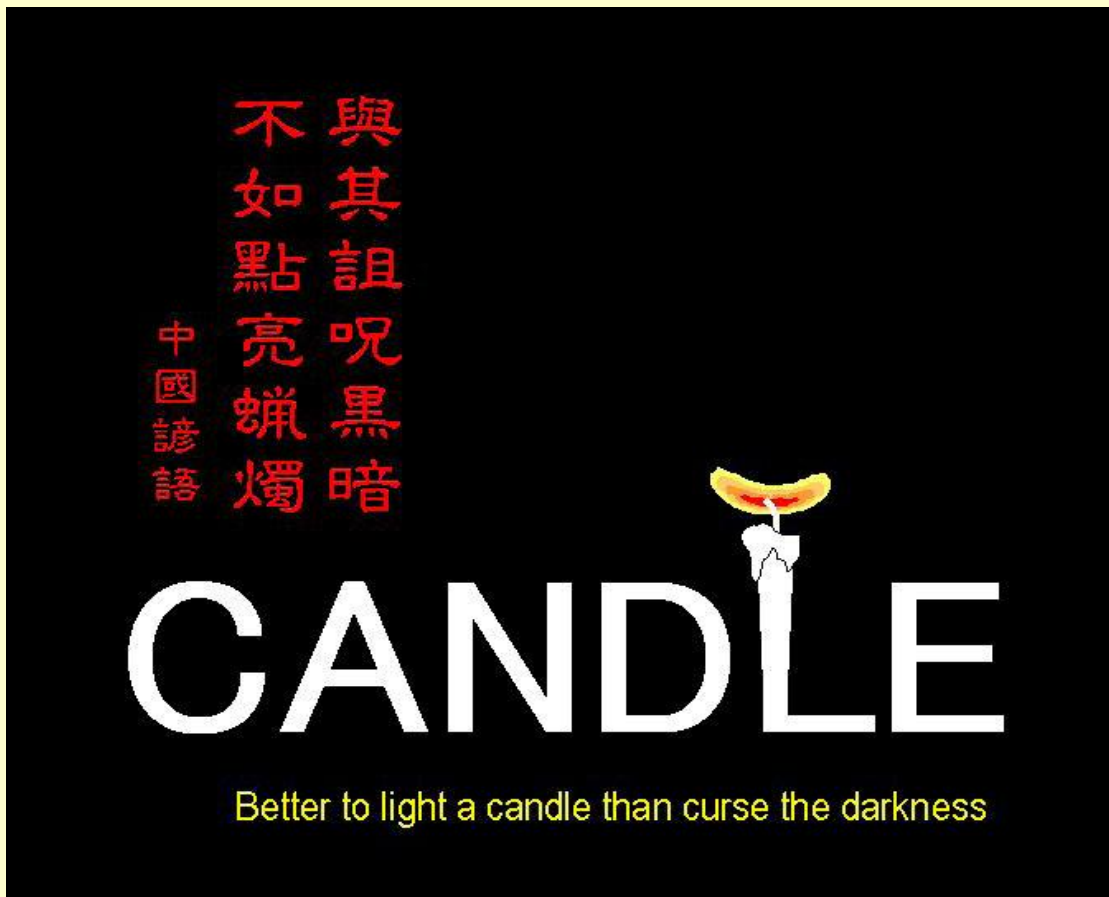
High-Reliability Afterheat-Dumping System

Система Съема Тепла



2001 Hiroshi Sekimoto (TIT, Japan) : CANDLE

Constant Axial Shape of Neutron Flux, Nuclide Densities and Power Shape During Life of Energy Production

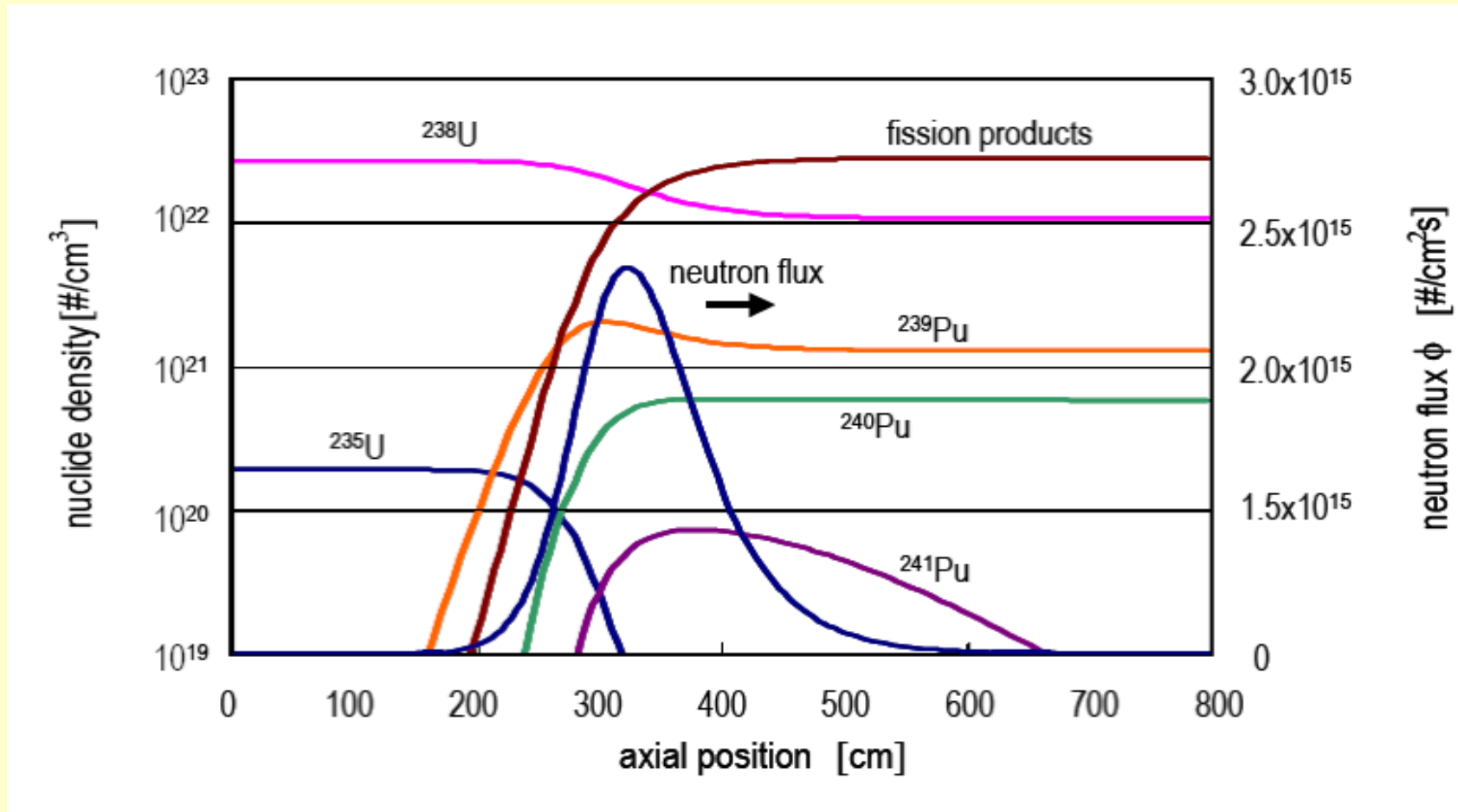


$$2d \quad x = z + Vt$$

Hiroshi Sekimoto et al. (TIT, Japan) 2001-2016 :

Stationary problem ($x = z + V t$), 2d cylindrical geometry (U-Pu fuel cycle)

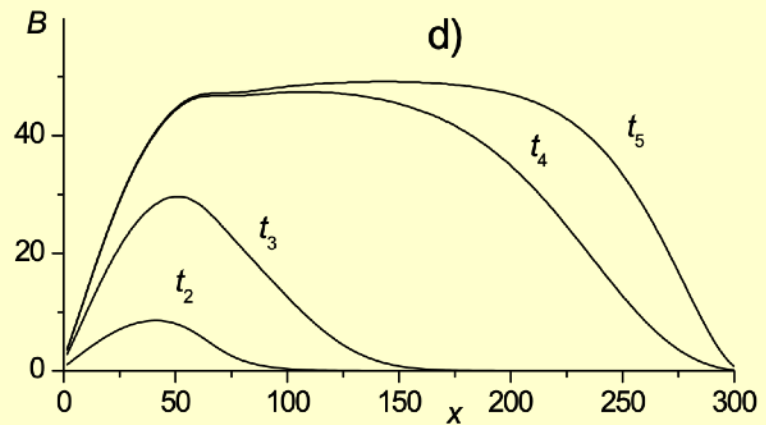
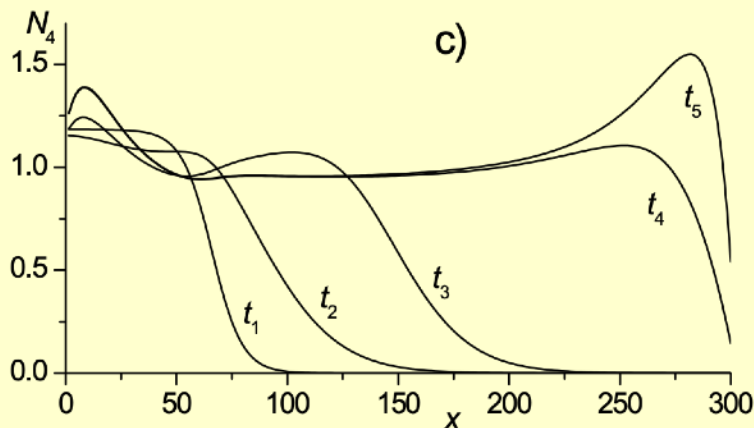
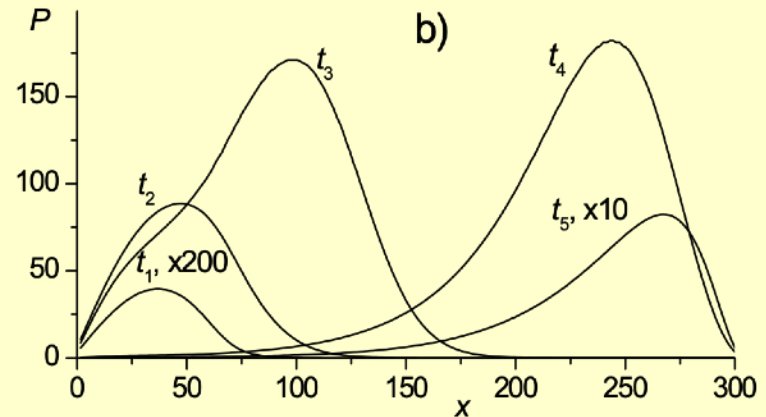
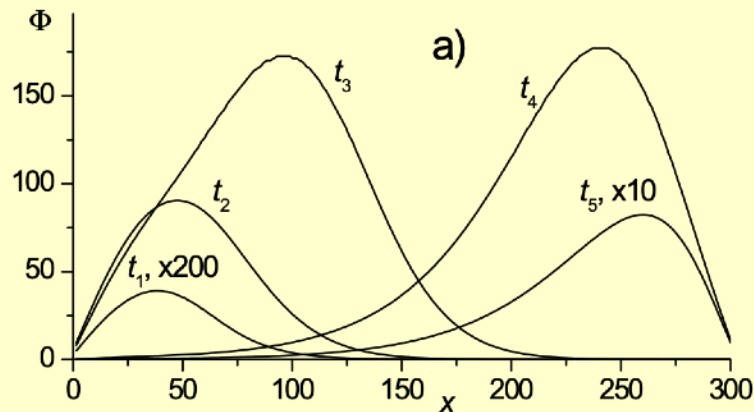
H. Sekimoto et al. *A New Burnup Strategy CANDLE*. Nuclear Science & Engineering 139 (2001) 306.



H. Sekimoto, *Light a CANDLE*, 2005, p.30 : **CANDLE burnup has not been achieved with thorium !?**

Nuclear burning wave in FR

S. Fomin et al., *Annals of Nuclear Energy*, 32 (2005) 1435-1456.



(a) scalar neutron flux ($\times 10^{16} \text{ cm}^{-2} \text{ s}^{-1}$);

(b) power density (kW cm^{-3});

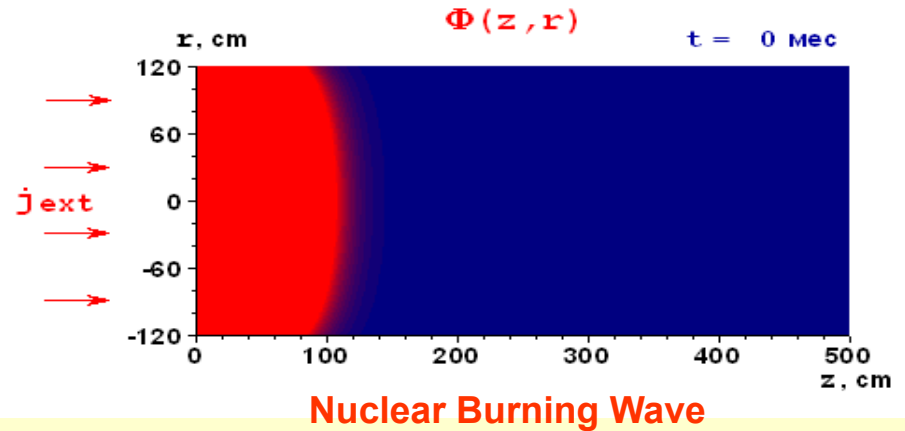
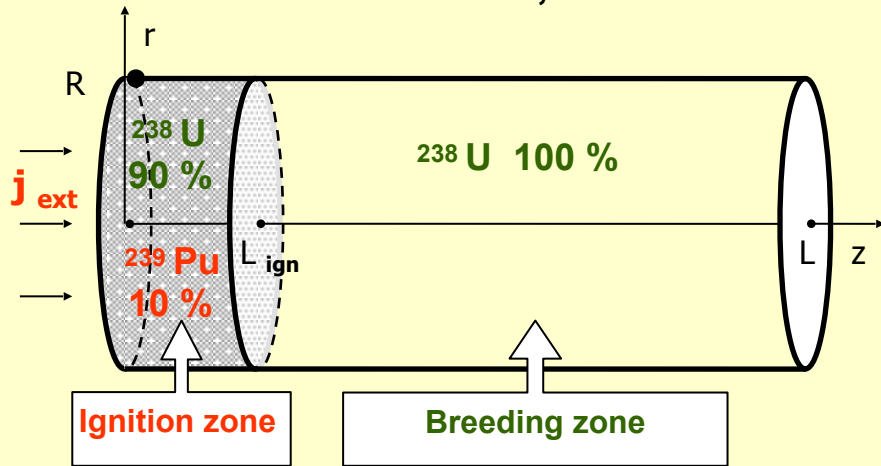
(c) concentration of ^{239}Pu ($\times 10^{21} \text{ cm}^{-3}$);

(d) depth of fuel burn-up (%)

for $t_1 = 1$, $t_2 = 80$, $t_3 = 100$, $t_4 = 140$ and $t_5 = 170$ days. ($0 \leq x \leq 300$ cm):

2D Non-Stationary Theory of Nuclear Burning Wave

S. Fomin, et al. - 1st IC "Global 2009", Paris, paper 9456.



Non-Stationary Nonlinear Multi-Group Diffusion Equation of Neutron Transport

$$\frac{1}{v^g} \frac{\partial \Phi^g}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} r D^g \frac{\partial \Phi^g}{\partial r} - \frac{\partial}{\partial z} D^g \frac{\partial \Phi^g}{\partial z} + \left(\Sigma_a^g + \Sigma_{in}^g + \Sigma_{mod}^g - \Sigma_{in}^{g \rightarrow g} \right) \Phi^g - \Sigma_{mod}^{g-1} \Phi^{g-1} =$$

$$= \chi_f^g \sum_{g'=1}^G (v_f \Sigma_f)^{g'} \Phi^{g'} - \sum_j \chi_d^j \sum_l \beta_l^j \sum_{g'=1}^G (v_f \Sigma_f)_l^{g'} \Phi^{g'} + \sum_j \chi_d^j \sum_l \lambda_l^j C_l^j + \sum_{g'=1}^{g-1} \Sigma_{in}^{g' \rightarrow g} \Phi^{g'}$$

Together with Fuel Burn-up Equations and Equations of Nuclear Kinetics

$$\frac{\partial N_l}{\partial t} = - \left(\sum_g \sigma_{al}^g \Phi^g + \Lambda_l \right) N_l + \left(\sum_g \sigma_{c(l-1)}^g \Phi^g + \Lambda_{(l-1)} \right) N_{(l-1)}, \quad (l=1+8); \quad \frac{\partial N_9}{\partial t} = \Lambda_6 N_6$$

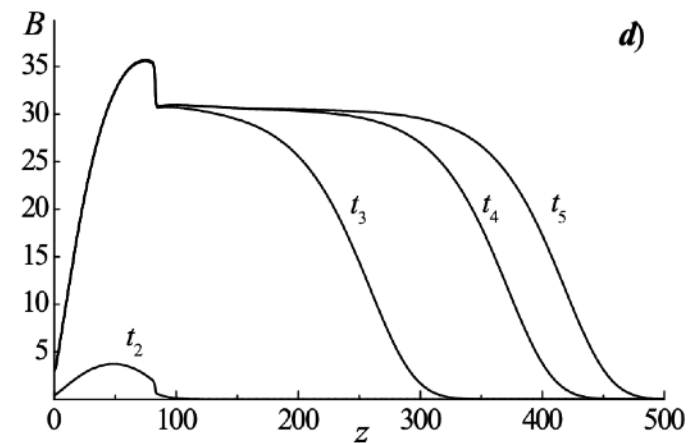
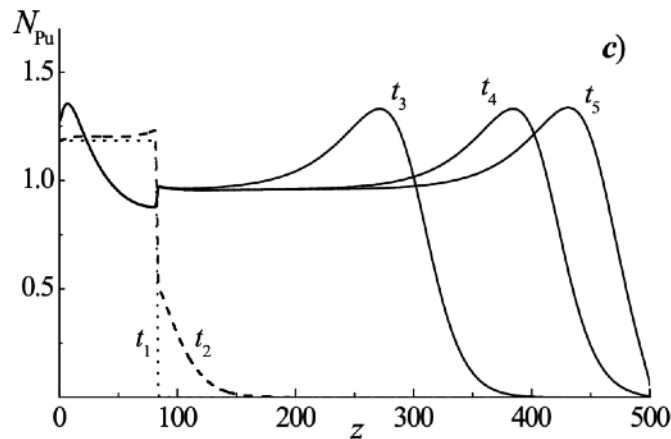
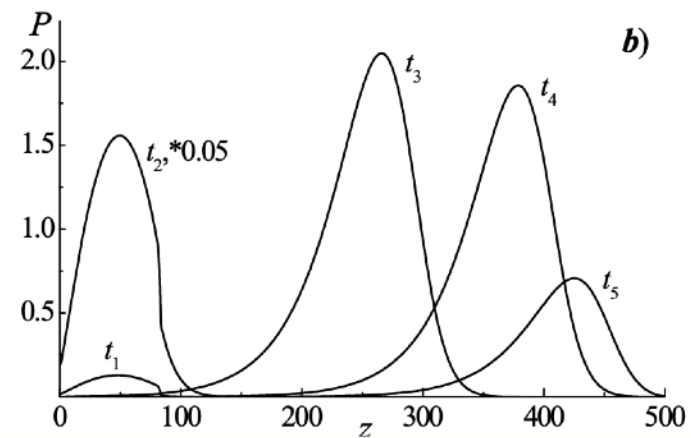
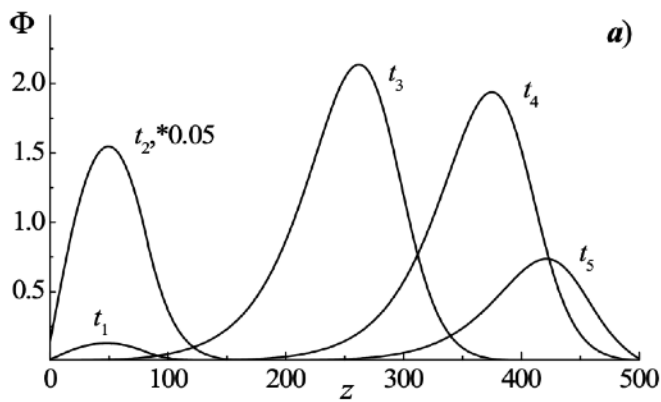
of Precursor Nuclei of Delayed Neutrons

$$\frac{\partial C_l^j}{\partial t} = -\lambda_l^j C_l^j + \beta_l^j \sum_g (v_f \Sigma_f)_l^g \Phi^g$$

$$\frac{\partial N_{10}}{\partial t} = \sum_{l=1,4,5,6,7} \left(\sum_g \sigma_{fl}^g \Phi^g \right) N_l$$

Metal fuel (44%)
 Pb-Bi coolant (36%)
 CM - Fe (20%)
 $j_{ext} \sim 10^{15} \text{ cm}^{-2} \text{ s}^{-1}$
 12
 $t_{off} = 400 \text{ days}$

Results for the 5m length and 110 cm radius cylindrical FR

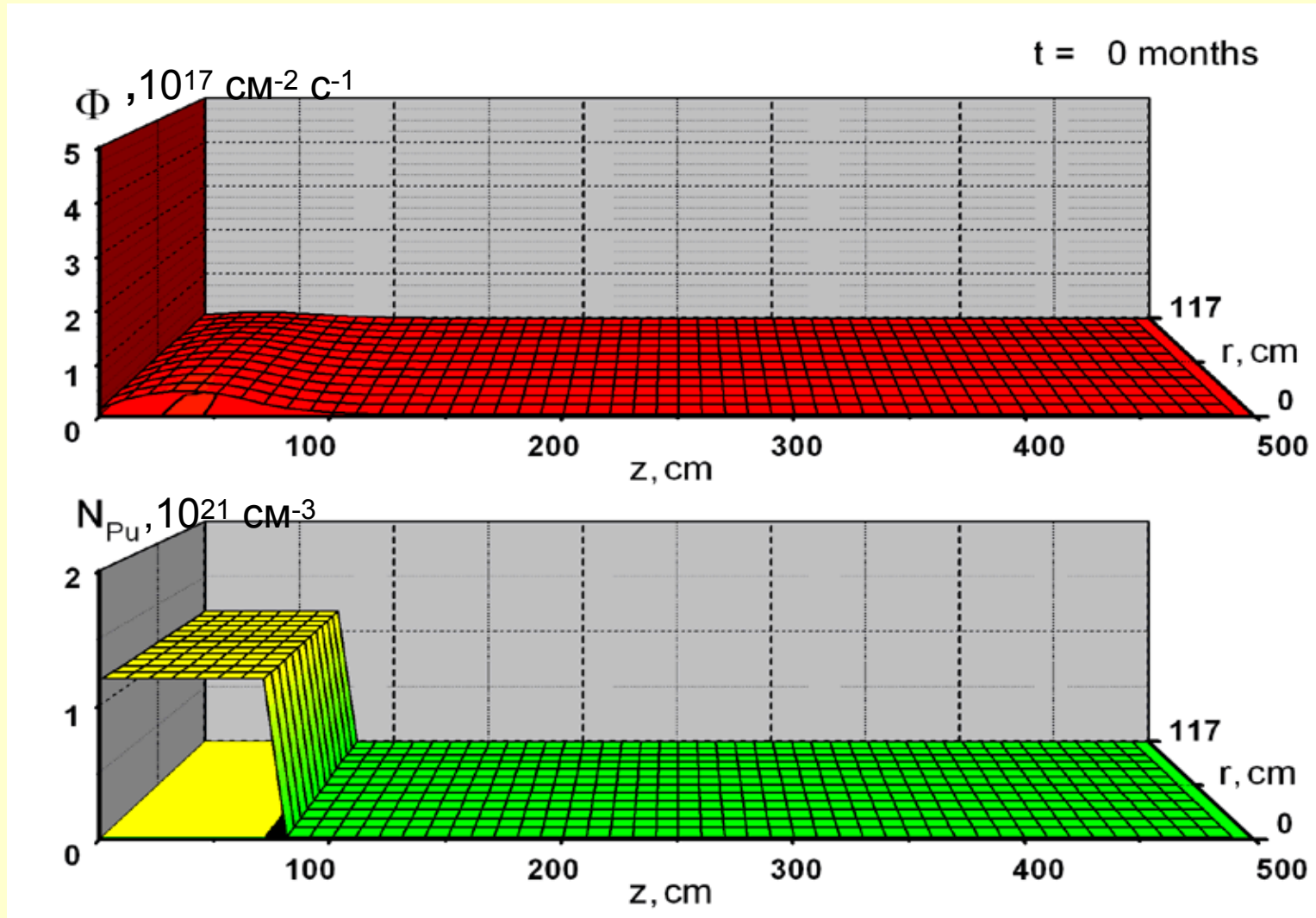


(a) scalar neutron flux ($\times 10^{16} \text{ cm}^{-2} \text{ s}^{-1}$); (b) power density (kW cm^{-3});

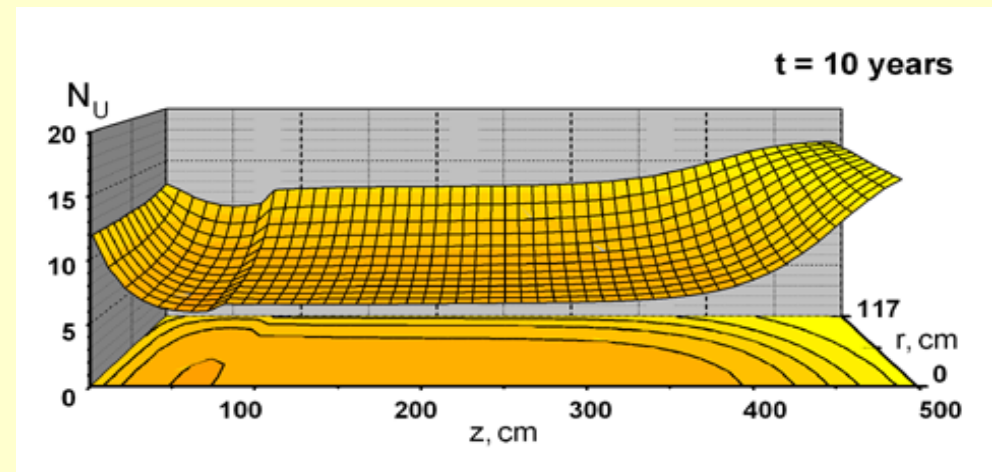
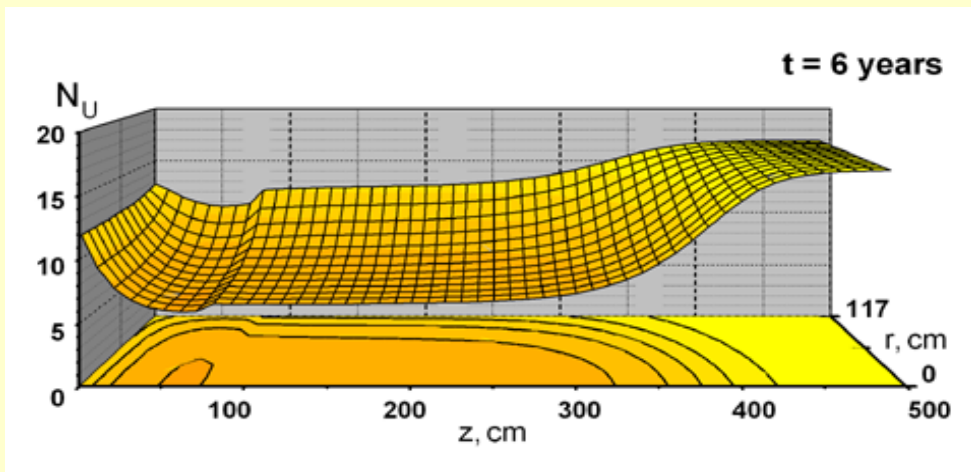
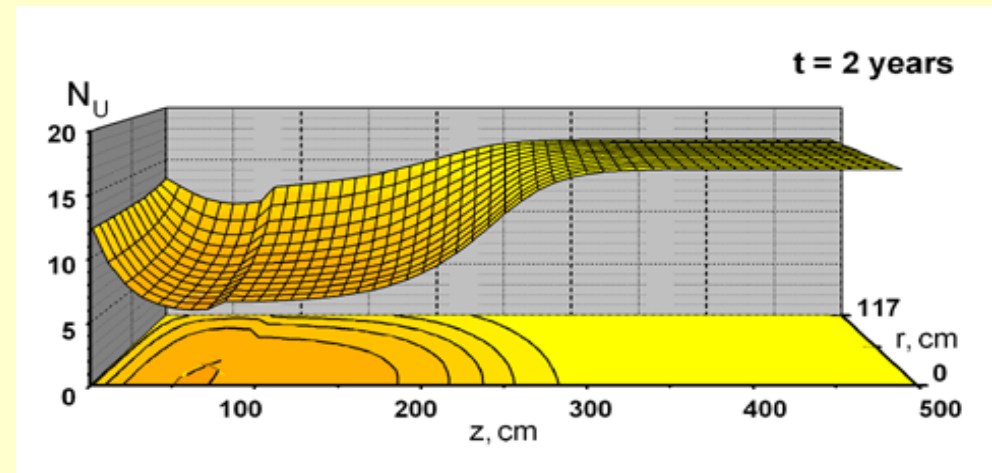
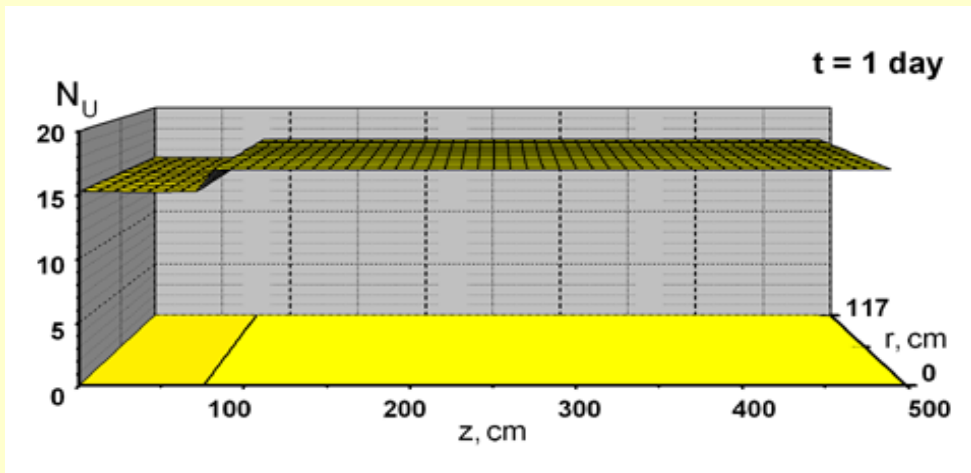
(c) concentration of ^{239}Pu ($\times 10^{21} \text{ cm}^{-3}$); (d) depth of fuel burn-up (%)

for for $t_1 = 5$, $t_2 = 100$, $t_3 = 2000$, $t_4 = 4000$ and $t_5 = 5000$ days.

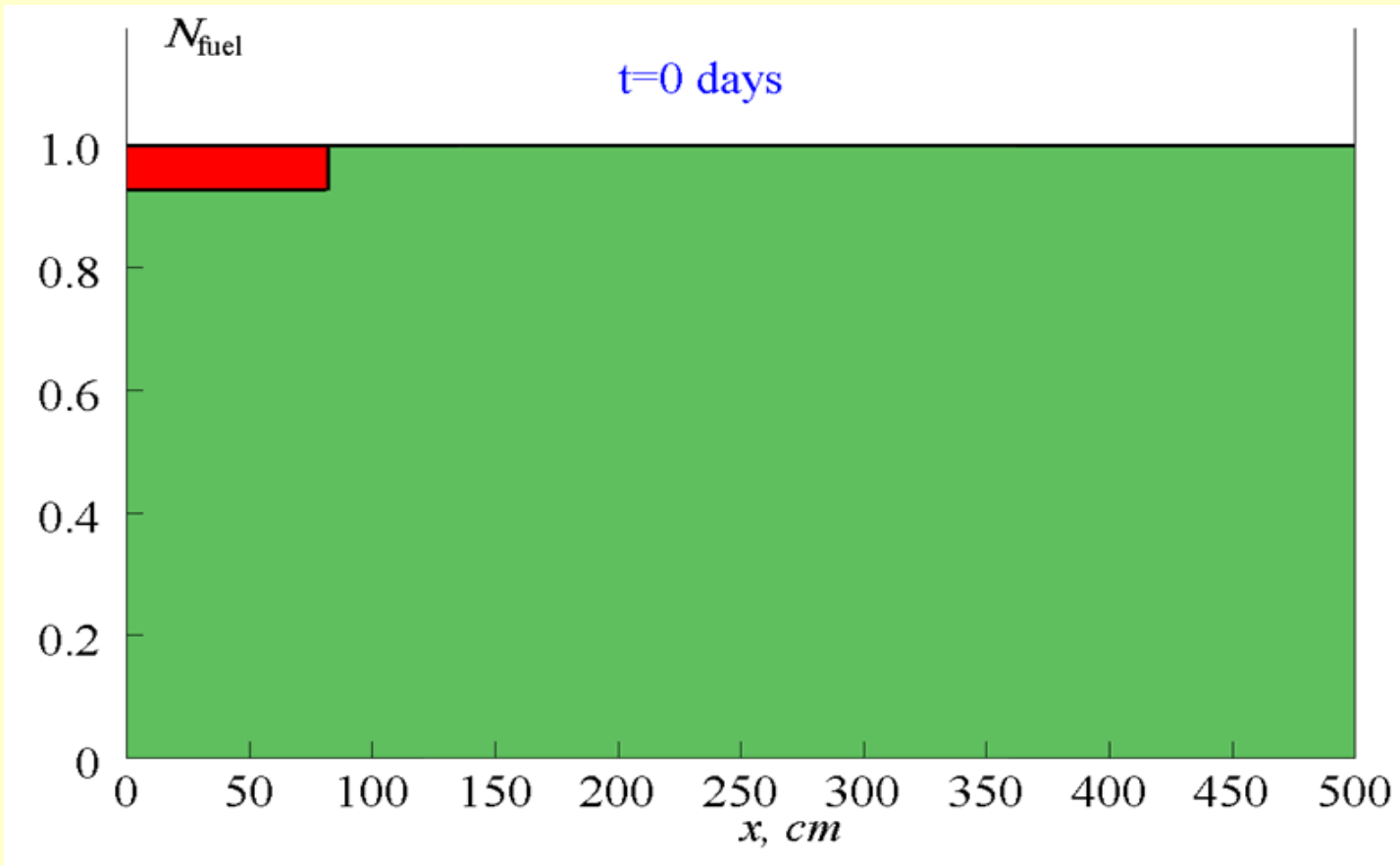
NBW Reactor : $R=117$ cm, $L = 500$ cm , $t_{\text{off}} = 950$ days



The 2D-distribution $N_U(r,z)$ ($\times 10^{21} \text{ cm}^{-3}$) of the ^{238}U isotope in the NBW regime at different time moments



Fuel burn-up



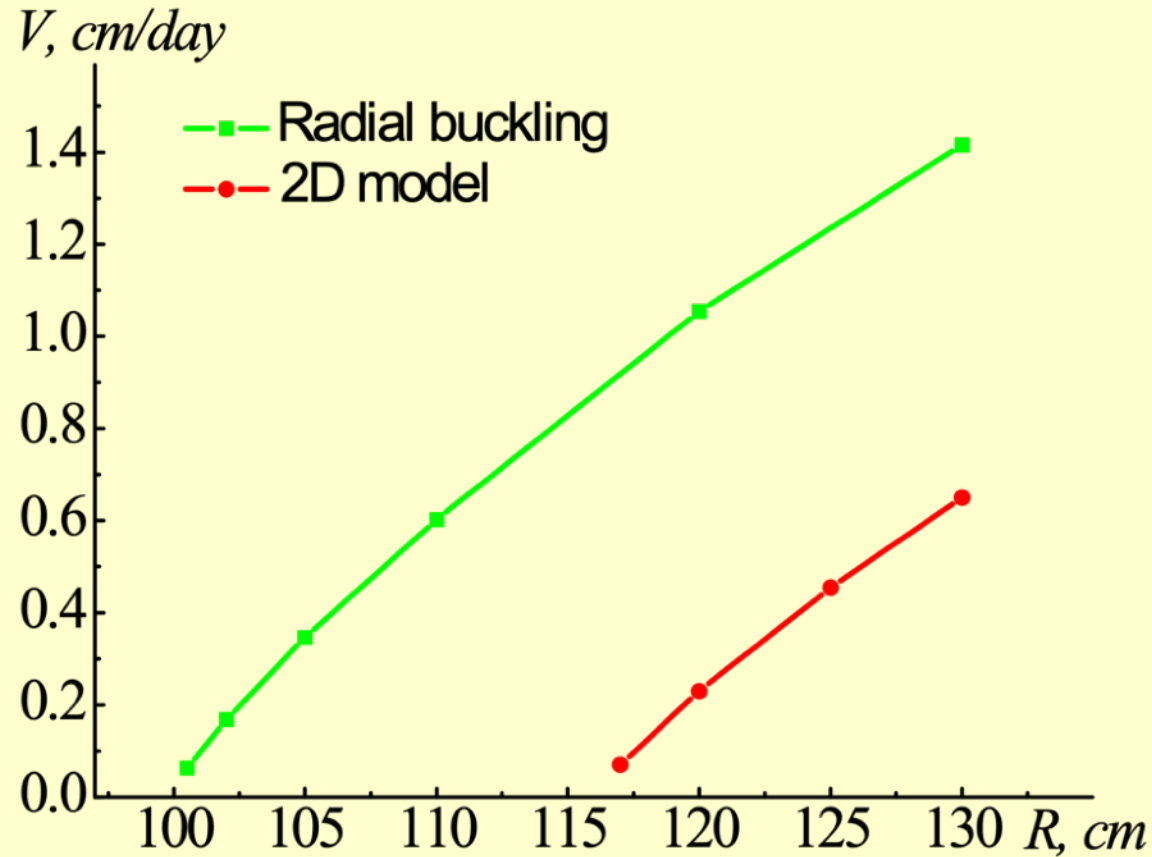
Fission products

^{239}Pu

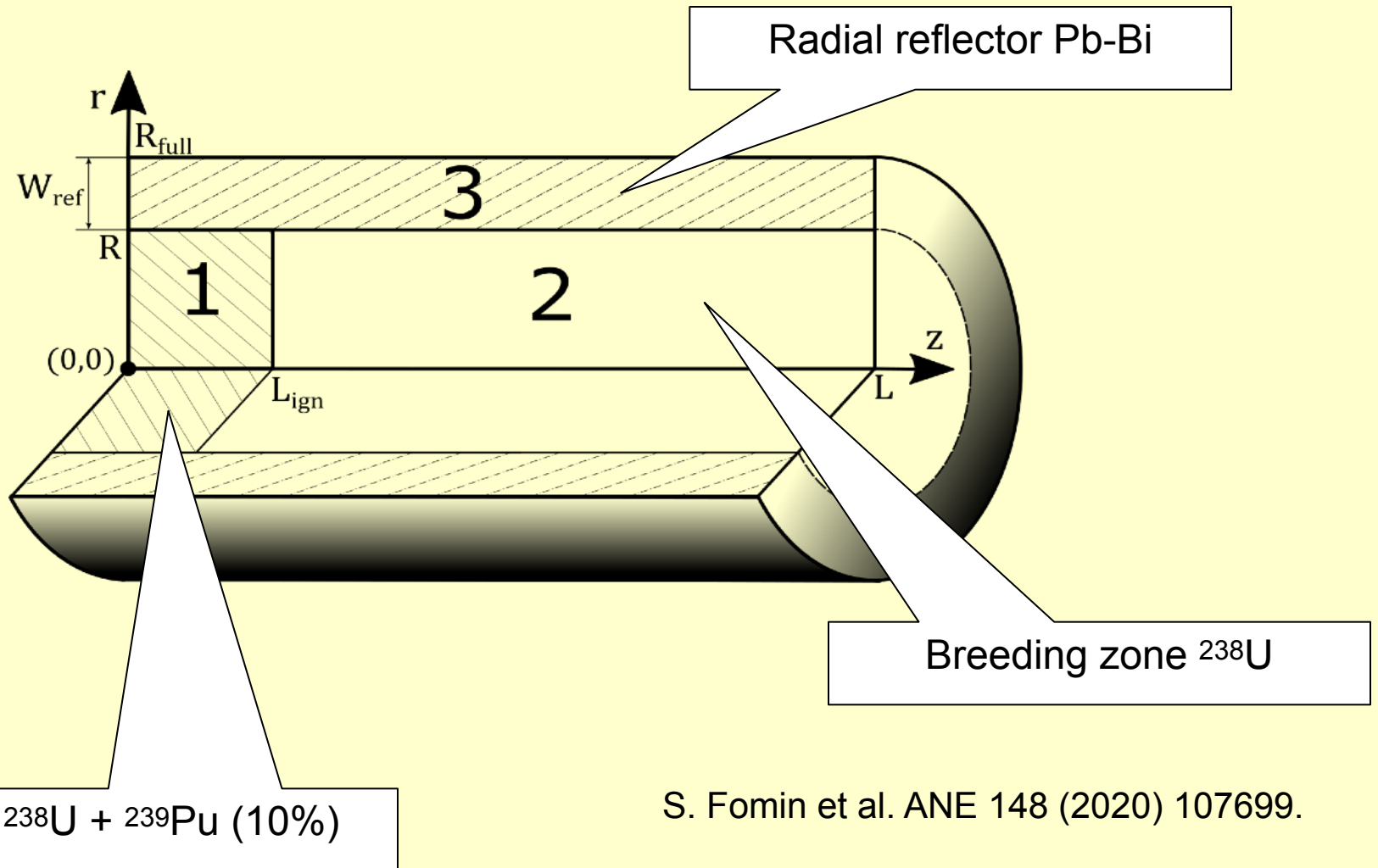
^{238}U

Dependence of the NBW velocity V on the reactor radius R

S. Fomin et al., **Global 2009** (Paris, France) [paper 9456](#)



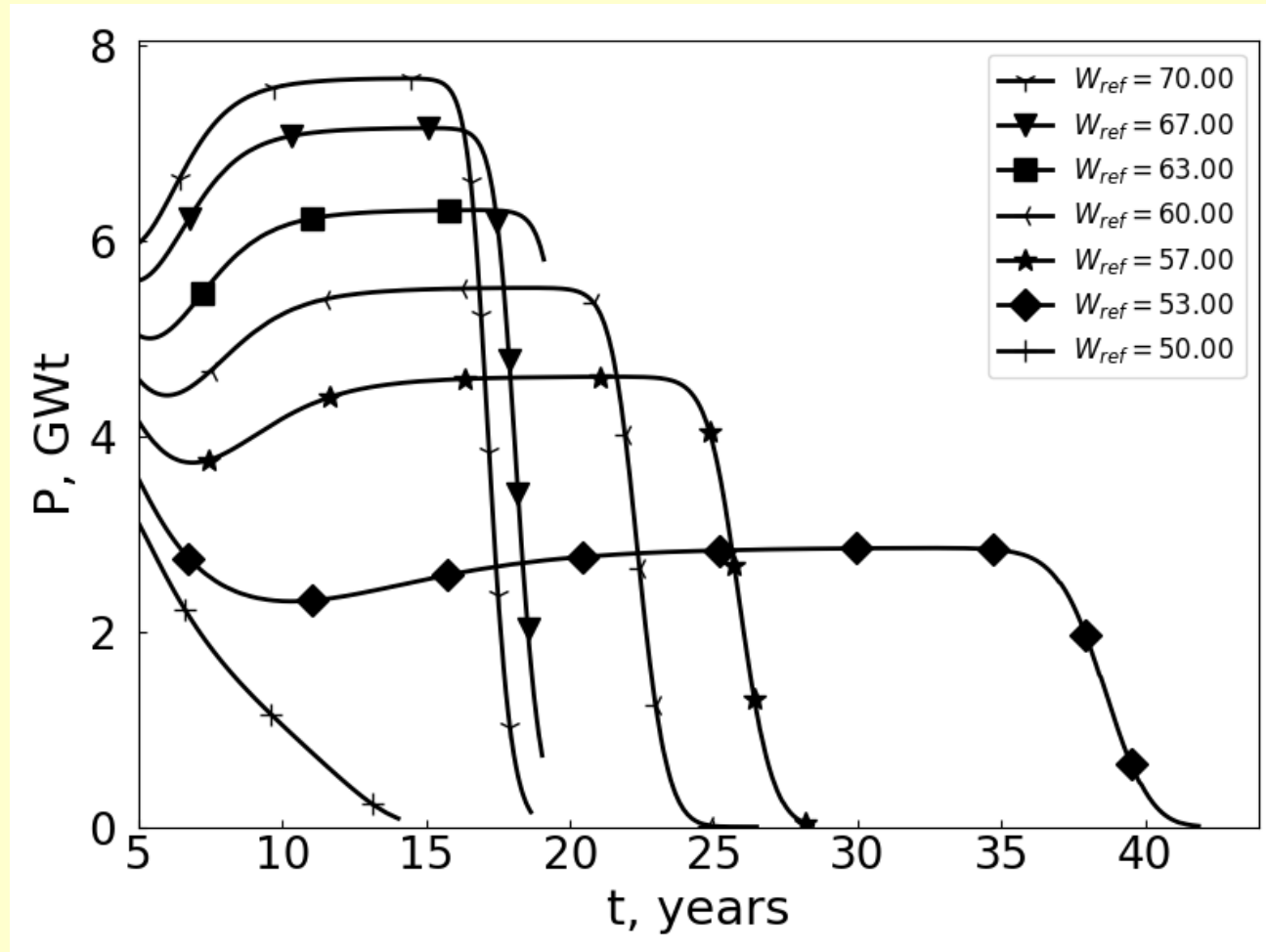
2D Non-Stationary Theory of Nuclear Burning Wave: The reflector effects study



S. Fomin et al. ANE 148 (2020) 107699.

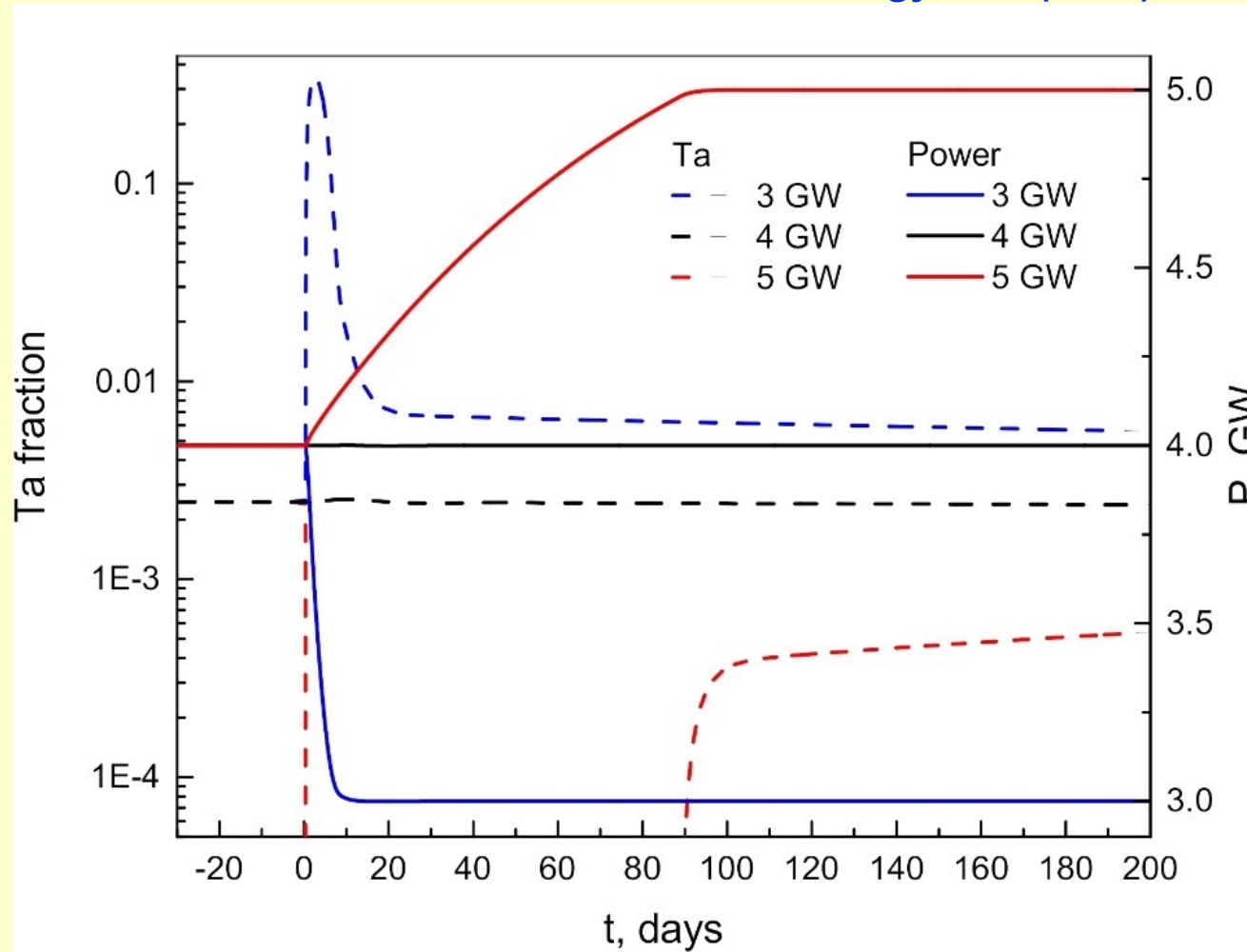
Reactor Power Control by Reflector Efficiency

S. Fomin et al., *Annals of Nuclear Energy*, 148 (2020) 107699.



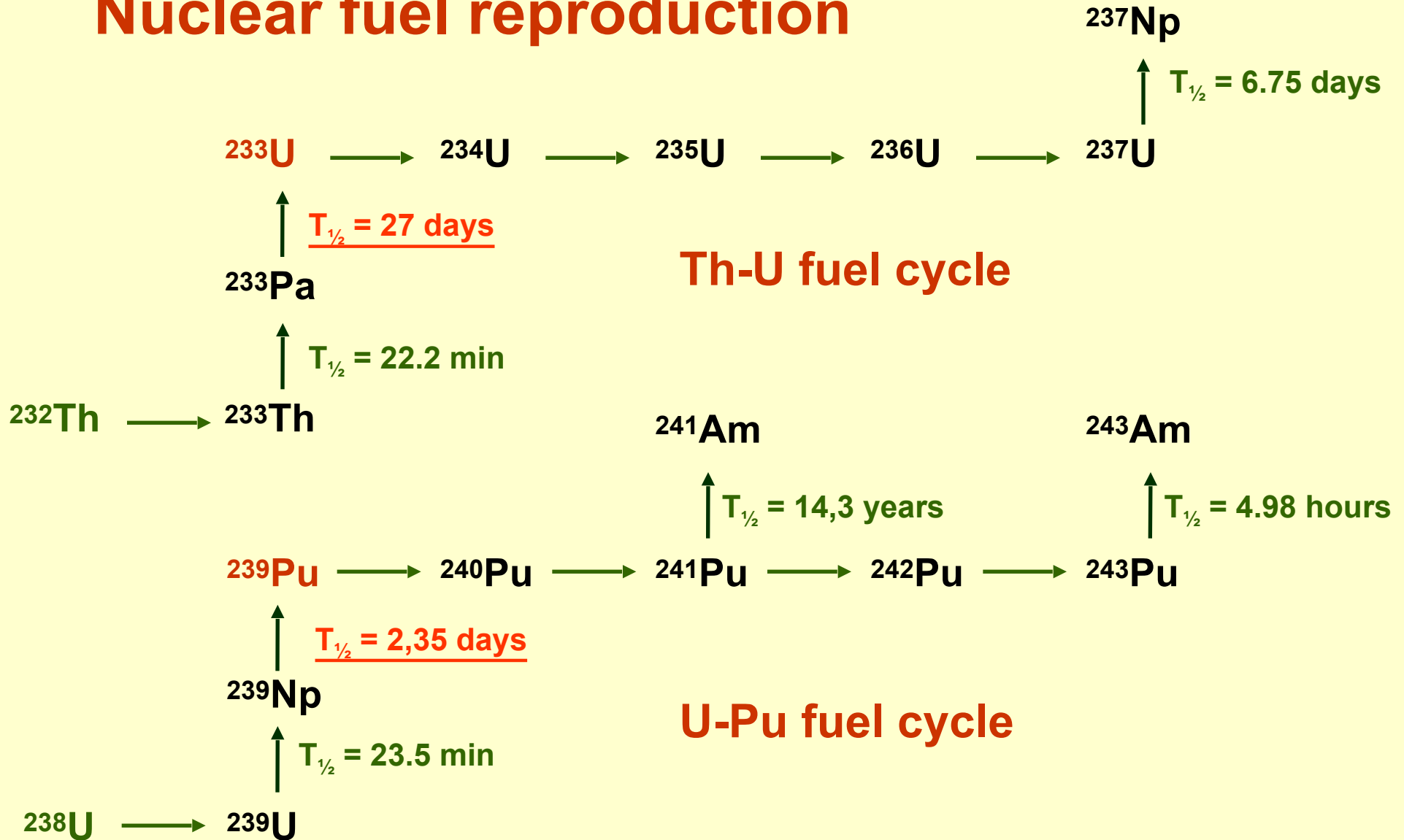
The reflector effects study

S. Fomin et al., *Annals of Nuclear Energy*, 148 (2020) 107699.



Malovytsia M.S., PhD thesis, 30 June 2021, Kharkiv, Ukraine.

Nuclear fuel reproduction



Dynamics of the FR nuclear composition

The numeration of the nuclei in the Th – U transformation chain

l	1	2	3	4	5	6	7	8	9	10
Nucleus	^{232}Th	^{233}Th	^{233}Pa	^{233}U	^{234}U	^{235}U	^{236}U	^{237}U	^{237}Np	FP

$$\frac{\partial N_1}{\partial t} = -\sigma_{a1}\Phi N_1 \quad \frac{\partial N_{10}}{\partial t} = \sum_{l=1,3\div 7,9} \sigma_{fl}\Phi N_l$$

$$\frac{\partial N_l}{\partial t} = -(\sigma_{al}\Phi + \Lambda_l)N_l + (\sigma_{c(l-1)}\Phi + \Lambda_{(l-1)})N_{(l-1)}, \quad (l = 2 \div 9)$$

$$\sigma_{al} = \sigma_{cl} + \sigma_{fl}, \quad \Lambda_l = \ln 2 / T_{1/2}^l, \quad N_l(z, t = 0) = N_{0l}(z)$$

Equations of nuclear kinetics for the precursor nuclei of delayed neutrons
(approximation of one equivalent group of delayed neutrons)

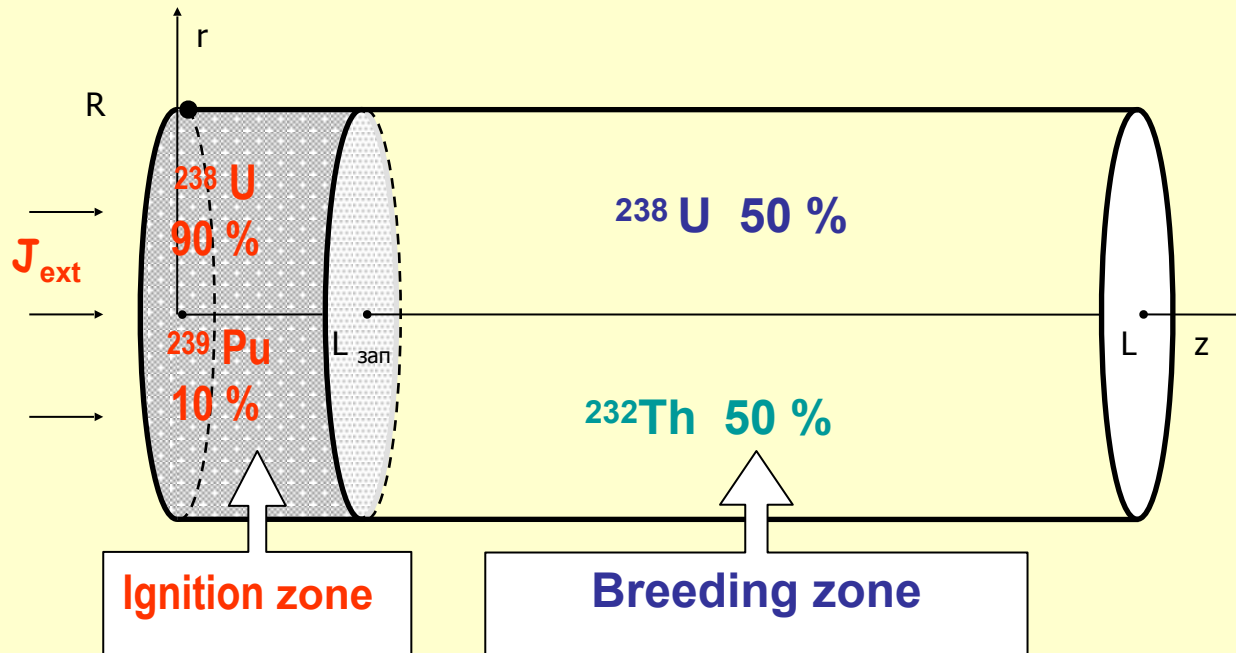
$$\frac{\partial C_l}{\partial t} = -\lambda_l C_l + \beta_l (\nu_f \Sigma_f)_l \Phi, \quad C_l(z, t = 0) = C_{0l}(z).$$

$l = 1, 3 \div 7, 9$ – the fissile nucleus number.

NBW reactor with mixed Th-U-Pu fuel

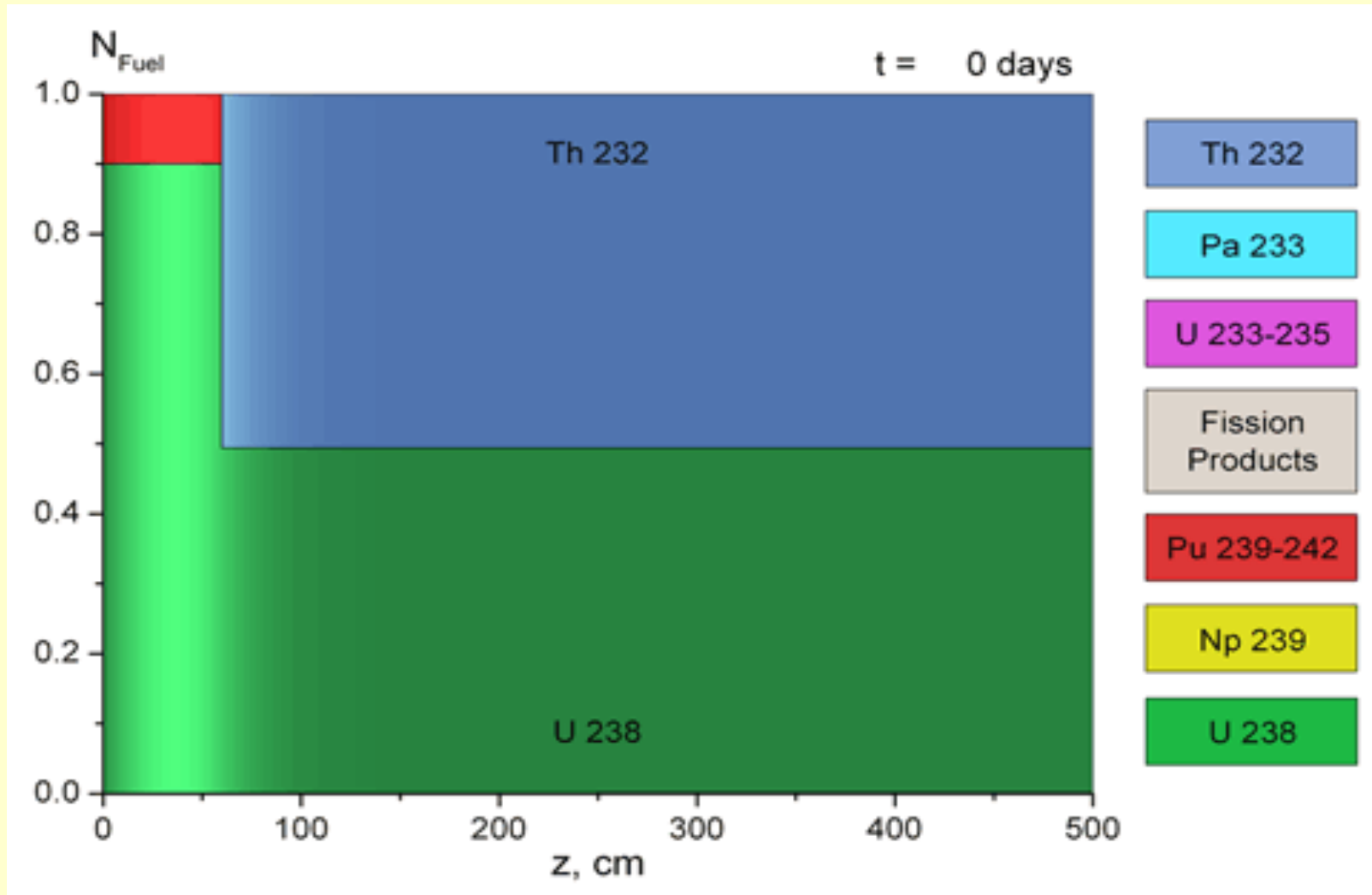
S. Fomin et al., ICAPP 2010 (San Diego, USA) paper 10302.

S. Fomin et al., Progress in Nuclear Energy, 52 (2011) 800-805.

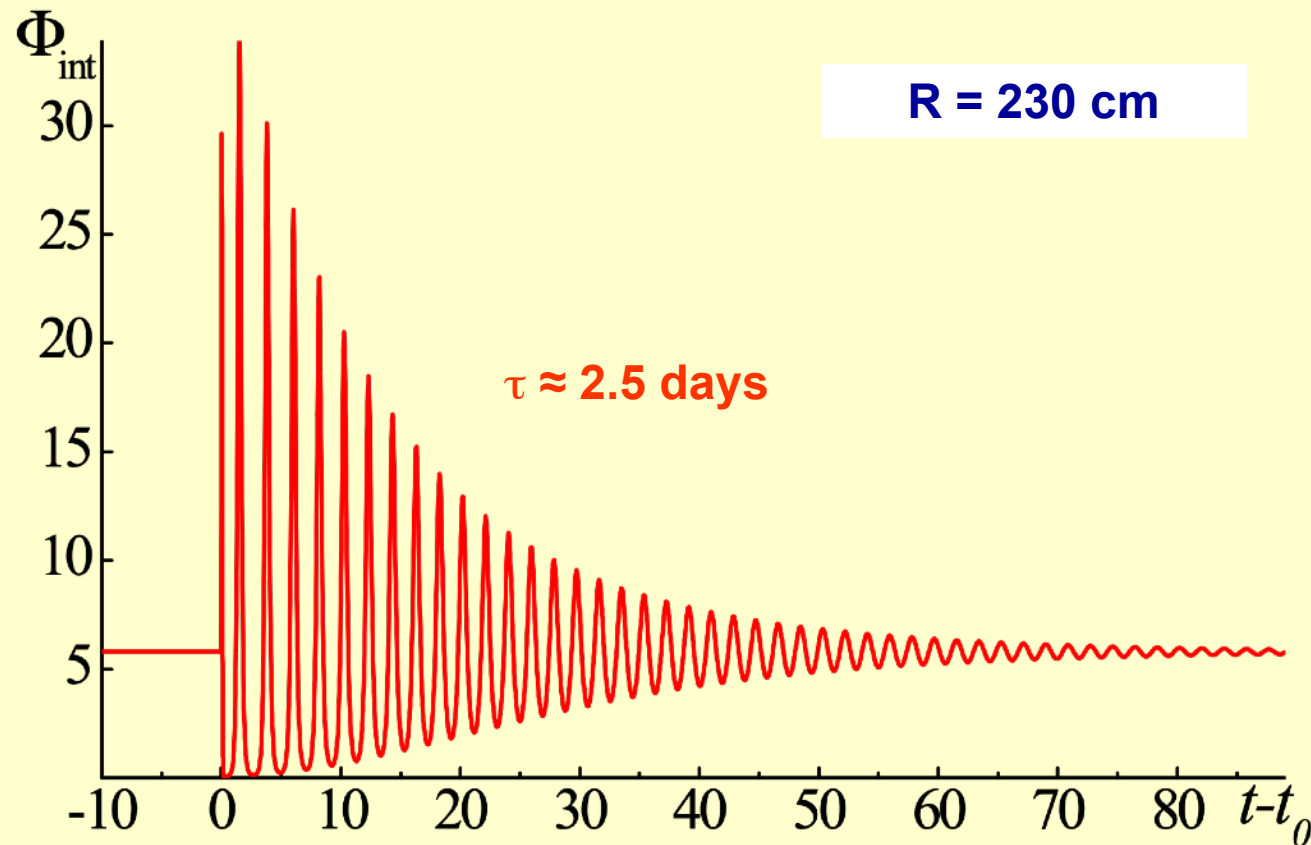


Example: Metallic fuel ^{232}Th (62%) + ^{238}U (48%) volume fraction = 55%,
fuel porosity $p = 0.35$; Coolant (Pb-Bi eutectic) vol. frac. = 30%,
Constr. materials (Fe) vol. frac. = 15%; $R = 390$ cm

Fuel burn-up for mixed Th-U-Pu cycle

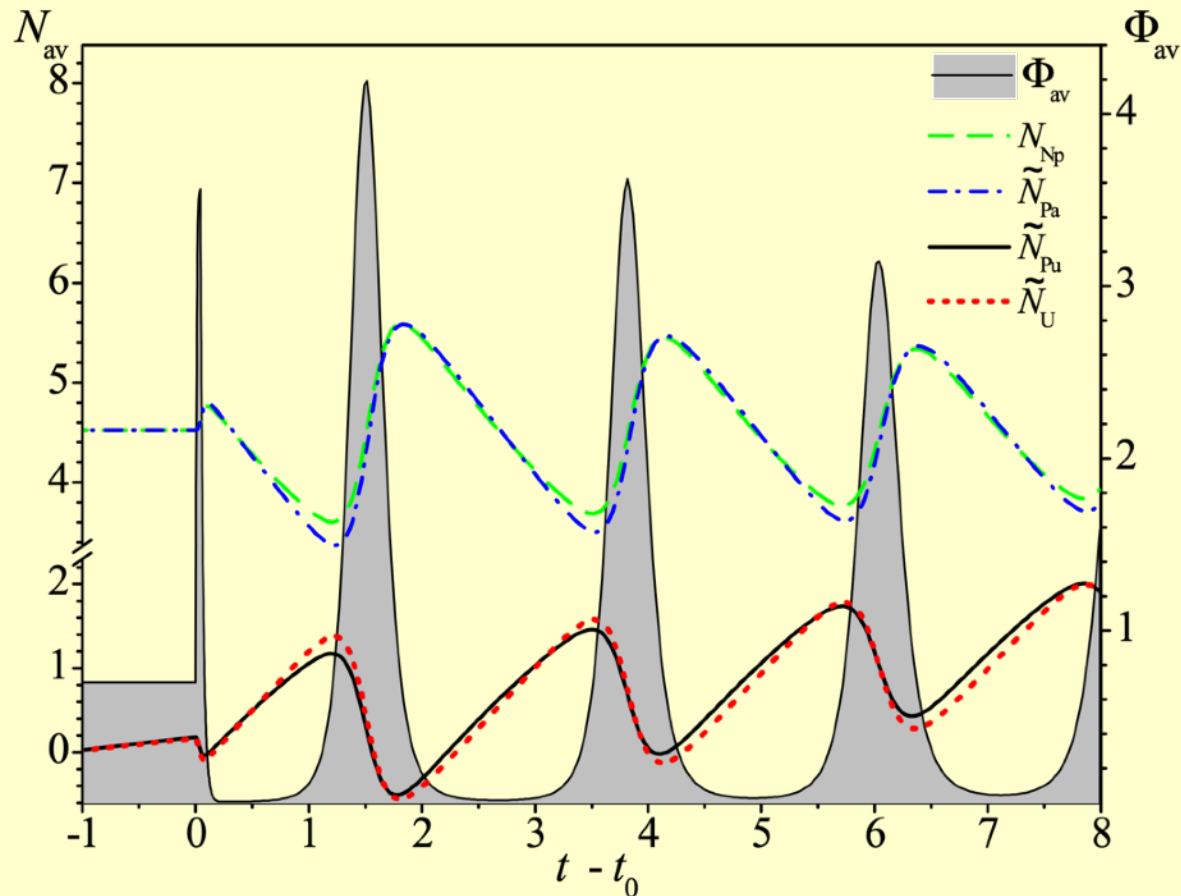


Stability of the NBW Regime



Perturbation of integral neutron flux F_{int} ($\times 10^{22} \text{ cm/s}$) caused by an external neutron source via time t (days). The source with intensity $Q_{\text{ext}} = 2 \times 10^{11} \text{ (cm}^{-3} \text{ s}^{-1}\text{)}$ starts at $t_0 = 3650$ days, lasts during 1 hour and is situated at $160 < z < 170 \text{ cm}$

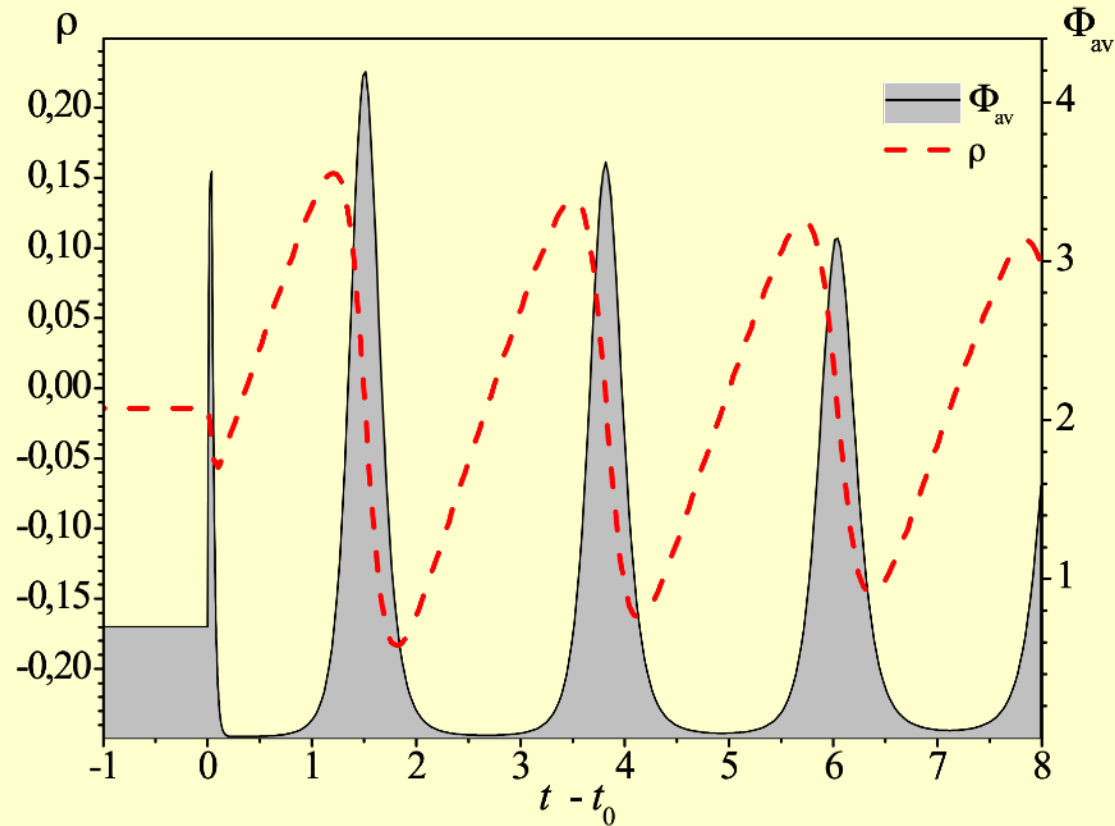
Negative Reactivity Feedback



$\tau \approx 2.5$ days

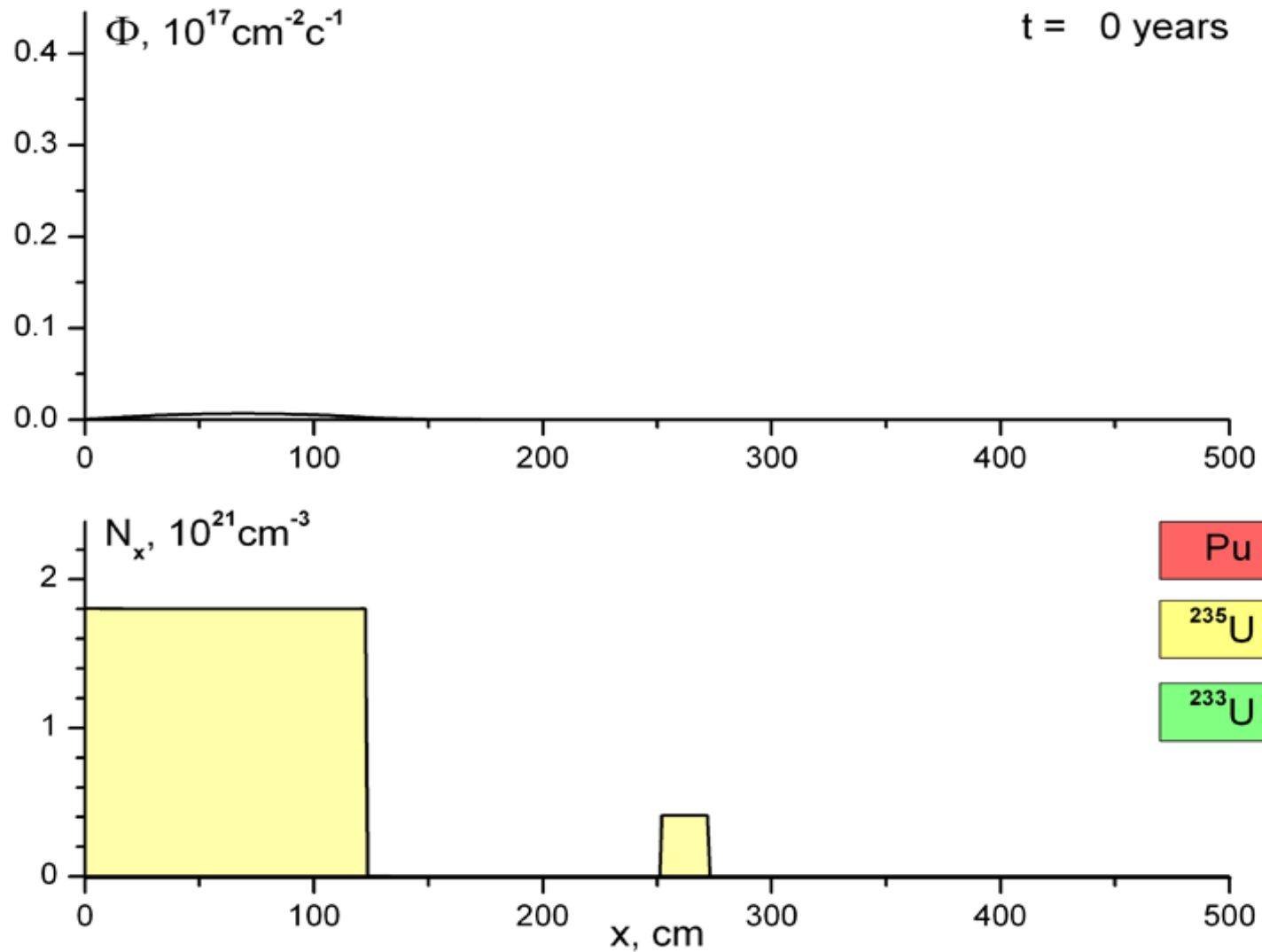
Evolution of the volume-averaged neutron flux F_{av} ($\times 10^{15} \text{ cm}^{-2} \text{ c}^{-1}$) and concentrations N_{av} ($\times 10^{17} \text{ cm}^{-3}$) of the main fissile and intermediate nuclides in the fuel of mixed ThUPu cycle with time t (days) at the initial stage of the neutron flux perturbation $t_0 = 3650$ days. The averaged nuclide concentrations: N_{Np} is for ^{239}Np , $N_{Pa} = N_{Pa} - 53.1 \cdot 10^{17} \text{ cm}^{-3}$, is for ^{239}Pu , $N_{Pu} = N_{Pu} - N_{Pu}|_{t_0-1}$ is for ^{233}U , $N_U = N_U - N_U|_{t_0-1}$

Negative Reactivity Feedback



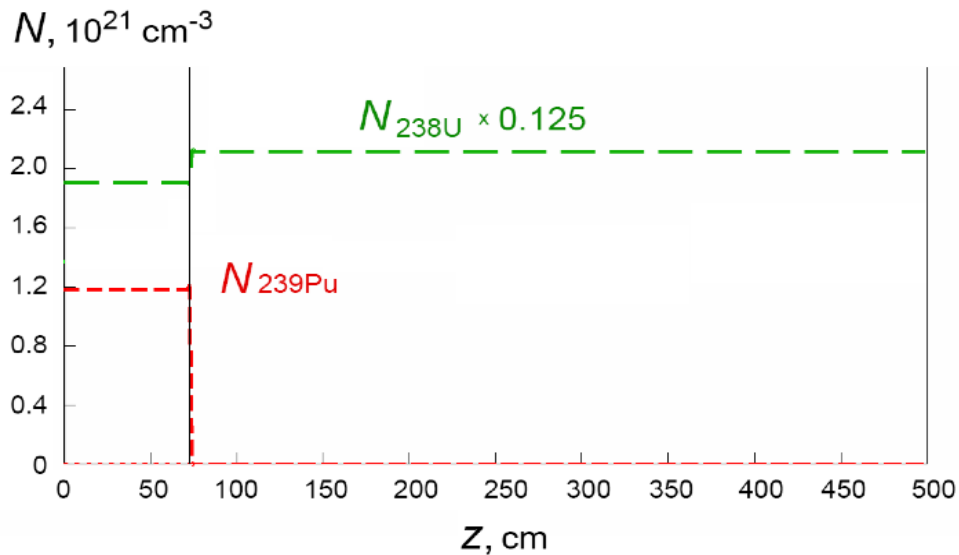
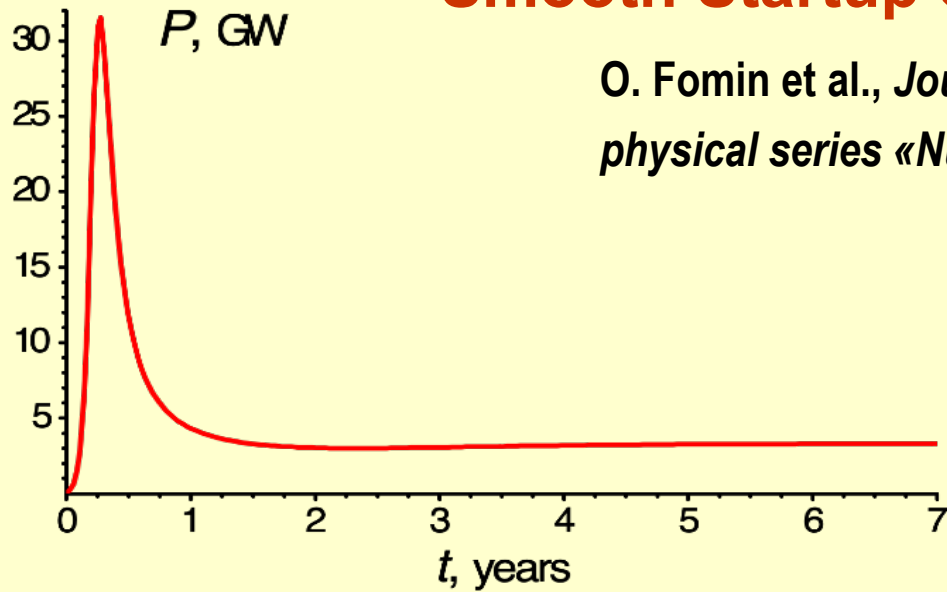
Variation of the reactivity ρ (dollars) with time t (days) along the variation of the volume-averaged neutron flux F_{av} ($\times 10^{15} \text{ cm}^{-2} \text{ c}^{-1}$)

Stability of the NBW Regime

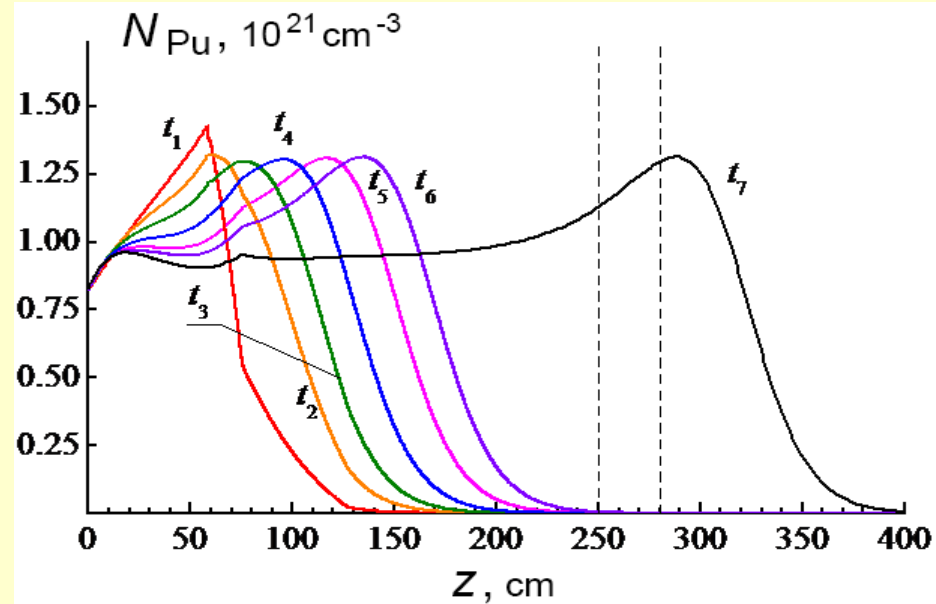
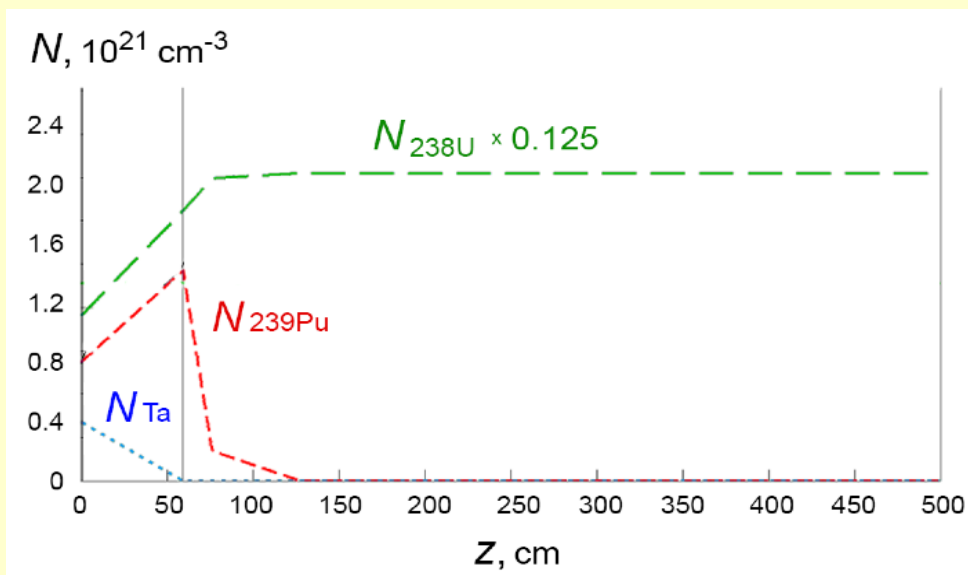
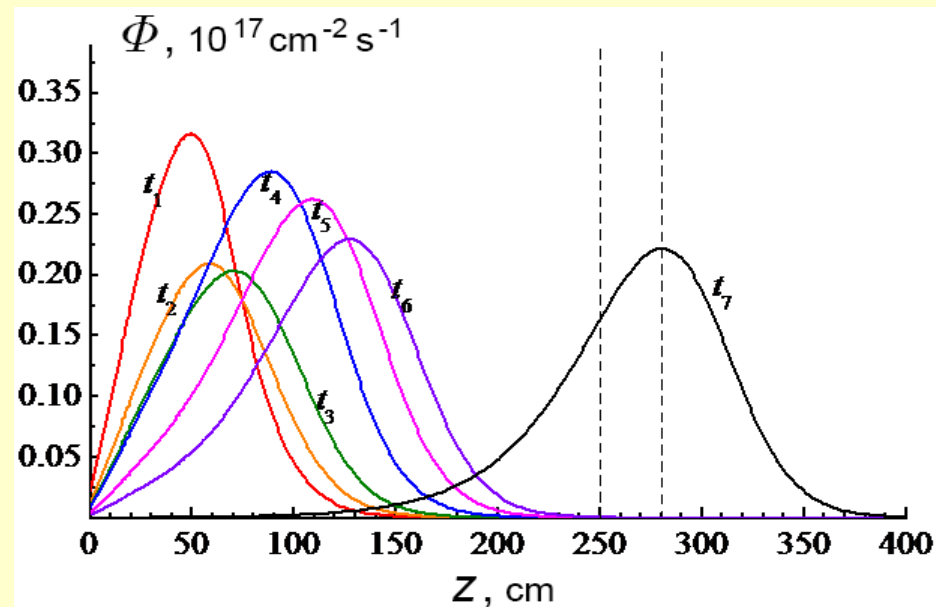
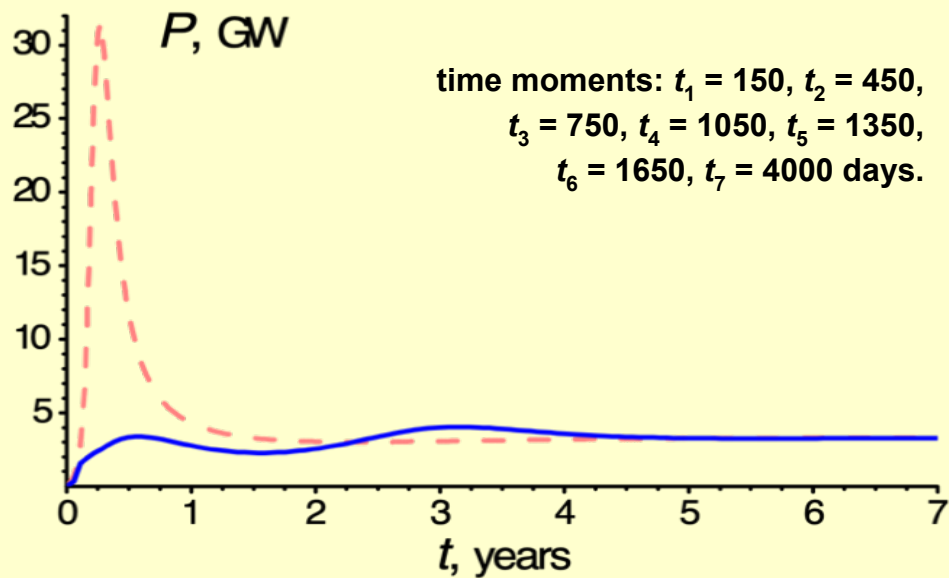


Smooth Startup of the NBW Reactor

O. Fomin et al., *Journal of Kharkiv National University*, #1041, physical series «Nuclei, Particles, Fields», 2 /58/ (2013) 49-56.

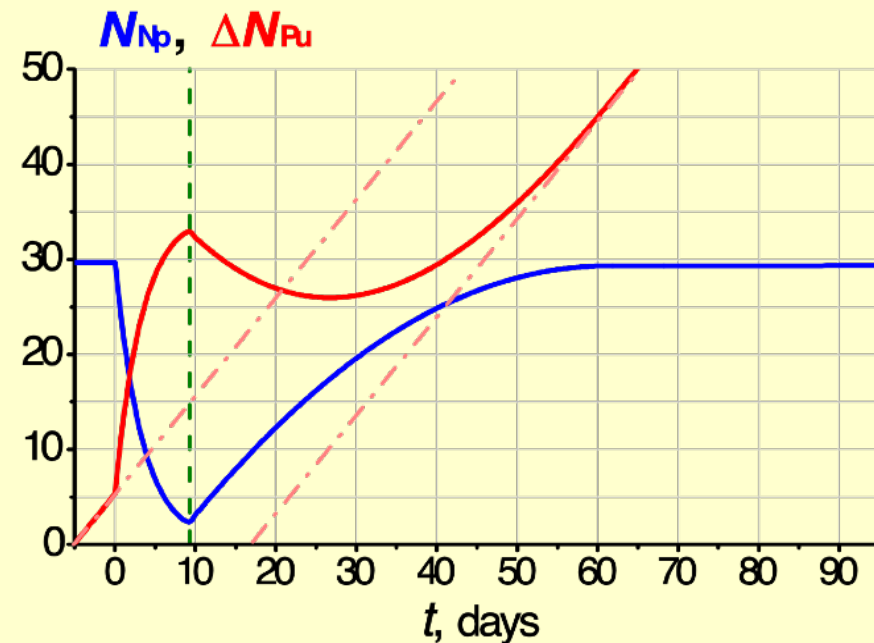
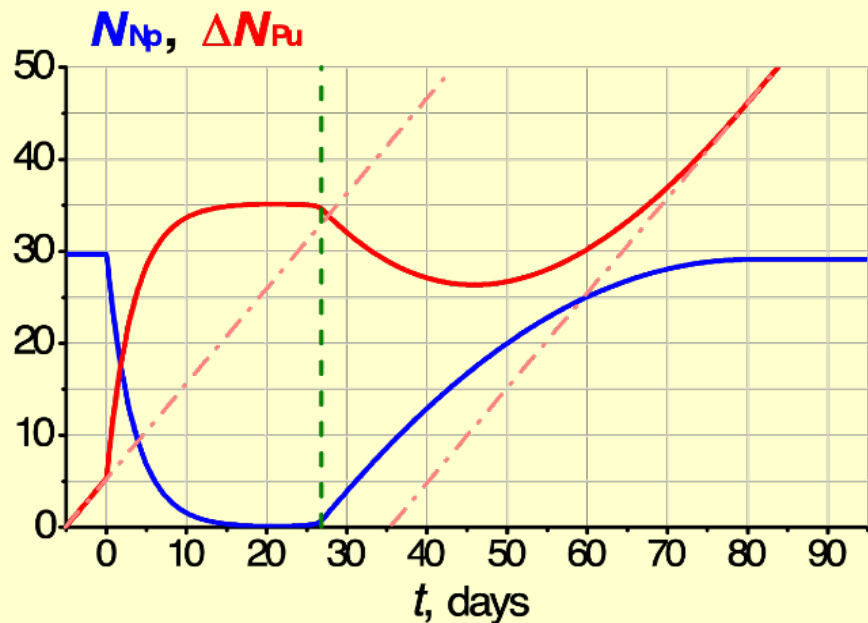
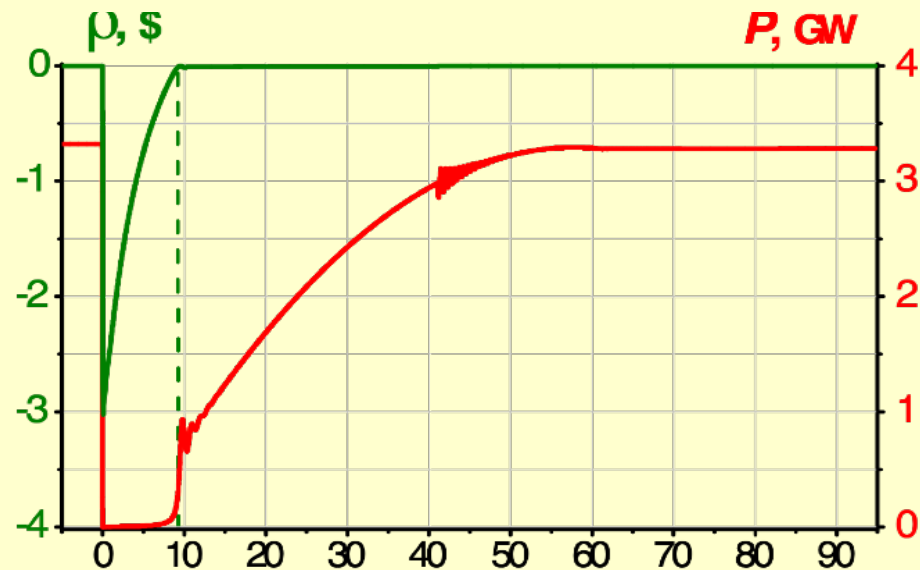
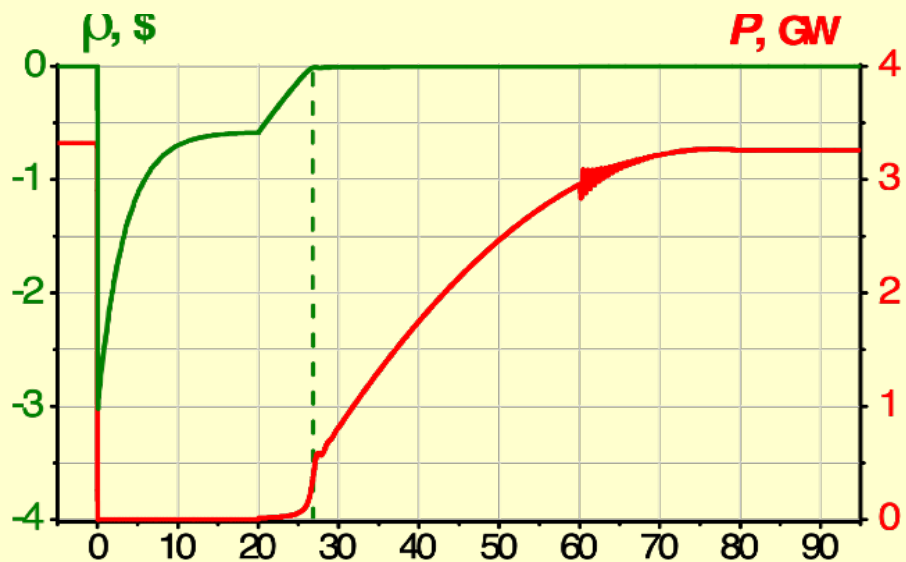


Smooth Startup of the NBW Reactor



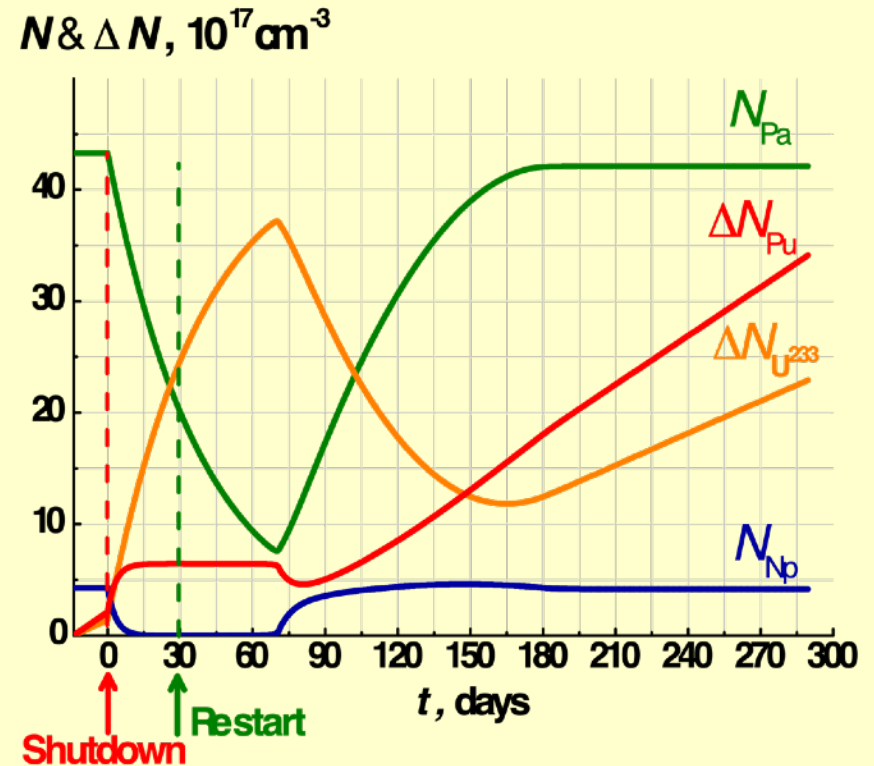
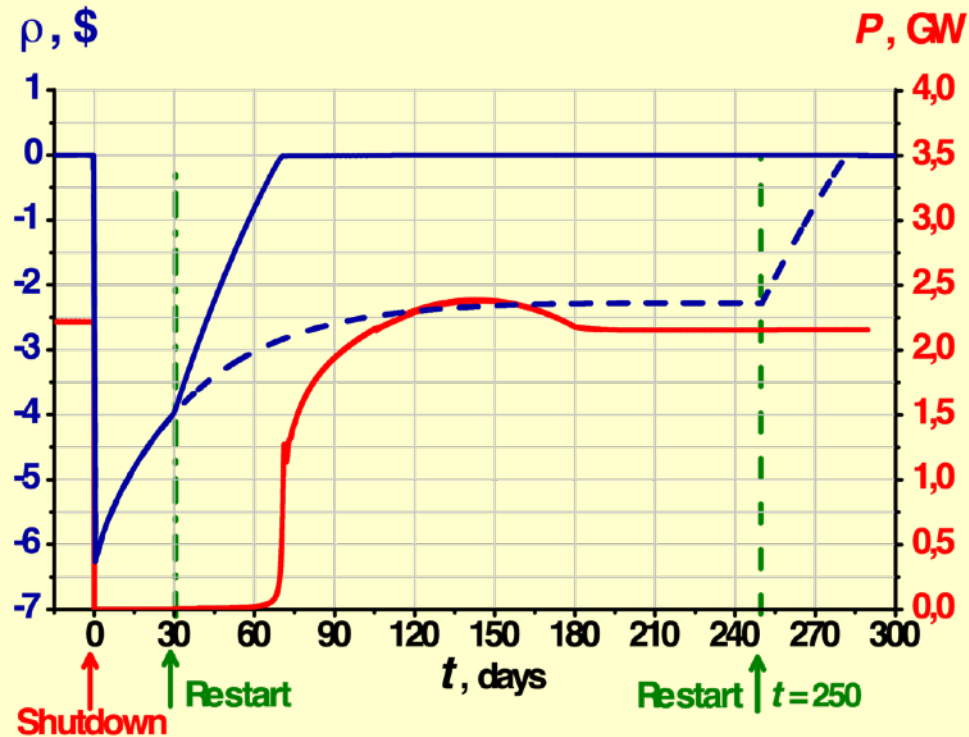
Shutdown and Restart of the NBW Reactor

S. Fomin et al., IC Global 2015 (Paris, France) paper 5254.



Shutdown and Restart of the NBW Reactor

S. Fomin et al., IC Global 2015 (Paris, France) paper 5254.



Main features of NBW reactor with mixed Th-U-Pu fuel cycle

Reactor composition (vol. frac.):

Fuel = 55% ($F_{\text{Th}} = 62\%$, $\rho = 0.20$), Coolant = 30%, CM = 15%, **R = 215 cm**

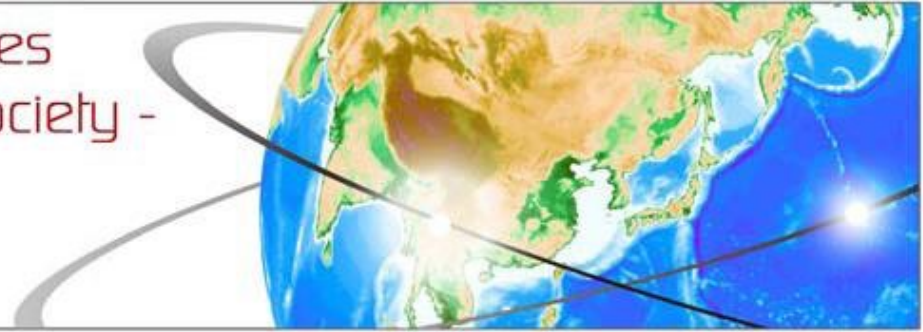
- **negative feedback on reactivity - intrinsic safety (!!!)**
- **long-term (decades!!) operation without refuelling and external control**
- **possibility of ^{232}Th and ^{238}U utilization as a fuel**
- **fuel burn-up depth for both ^{238}U and $^{232}\text{Th} \approx 50\%$ (one through cycle !)**
- **neutron flux in active zone $\approx 2 \cdot 10^{15}$ n/cm²s**
- **neutron fluence during the whole reactor campaign $\approx 3 \cdot 10^{24}$ n/cm²**
- **energy production density in active zone ≈ 200 W/cm³**
- **total power at the steady-state regime ≈ 1.2 GWt**
- **wave velocity at the steady-state regime ≈ 2 cm/year**
- **possibility of nuclear waste burn out (expected)**

List of our publications on the NBW reactor :

- S. Fomin et al., *Annals of Nuclear Energy*, 32 (2005) 1435-1456.
- S. Fomin et al., *Problems of Atomic Science & Technology*, 6 (2005) 106-113.
- S. Fomin et al., ICENES (2005) (Brussels, Belgium) paper IC058.
- S. Fomin et al., *Nuclear Science & Safety in Europe*. Springer (2006) 239-251.
- S. Fomin et al., ICAPP'06 (2006) (Reno, USA) paper 6157.
- S. Fomin et al., *Problems of Atomic Science & Technology*, 3 (2007) 156–163.
- S. Fomin et al., ICAPP'07 (2007) (Nice, France) paper 7499.
- S. Fomin, *Reactor Physics and Technology*. PINP WS, St-Petersburg, XL-XLI (2007) 154-198.
- S. Fomin et al., *Progress in Nuclear Energy*, 50 (2008) 163-169.
- Yu.Mel'nik et al., *Atomic Energy*, 107 (2009) 288-295.
- S. Fomin et al., *Global 2009* (Paris, France) paper 9456.
- S. Fomin et al., *ICAPP 2010* (San Diego, USA) paper 10302.
- S. Fomin et al., *Progress in Nuclear Energy*, 52 (2011) 800-805.
- O. Fomin et al., *Journal of KNU*, #104, «Nuclei, Particles, Fields», issue 2 /58/ (2013) 49-56.
- S. Fomin et al., IC “Fast Reactors 2013” (Paris, France) paper CN-199-457.
- S. Fomin et al., IC “Global 2015” (Paris, France) paper 5254.
- S. Fomin et al., *Problems of Atomic Science & Technology*, 3 /121/ (2019) 80–85
- S. Fomin et al., *Annals of Nuclear Energy*, 148 (2020) 107699

- Innovative Nuclear Technologies
for Low-Carbon Society -

31st October – 3rd November, 2010
Tokyo Institute of Technology, Tokyo, Japan



1A-1-2: Sustainable Burning Reactors - Chairs: Kevan Weaver (TerraPower, USA)

Traveling-Wave Reactors: Challenges and Opportunities - Kevan Weaver et al. (TerraPower, USA)

Feasibility of LBE Cooled Breed and Burn Reactors - Ehud Greenspan (UC, Berkeley, USA)

Preliminary Engineering Design of Sodium-Cooled CANDLE Core - Hiroshi Sekimoto (TIT, Japan)

Nuclear Burning Wave in Fast Reactor with Mixed Th-U Fuel - Sergii Fomin et al (NSC KIPT, Ukraine)

Nuclear Traveling Wave in a Supercritical Water Cooled Fast Reactor – W. Maschek (KIT, Germany)

Development and Prospects of TWR Project in China - Zheng Mingguang (Shanghai NER&DI, China)

Special Presentation: Traveling-Wave Reactors - John Gilliland. (Director of TerraPower, USA)

1A-3: Thorium Fuel Reactors - Chair: Sergii Fomin (NSC KIPT, Ukraine)

(Th-U-Pu) - Mixed Fuel Cycle and Proliferation– E. Kryuchkov et al, (MEPhI, Russia)

Large Scale Utilization of Thorium in Gas Cooled Reactors - V. Jagannathan (Bhabha ARC, India)

...

Program: HORIZON-EURATOM-2023-NRT-01-03

Proposal acronym: **"TREASURE"** (4 MEuro for 4 years)

Subject: Development of the European Generation IV Gas-cooled Fast Reactor (demonstrator) "ALLEGRO"

List of participating organisations from 8 countries:

- 1 VUJE AS **SK** (Coordinator)
- 2 UJV REZ AS **CZ**
- 3 ENERGIATUDOMANYI KUTATOKOZPONT **HU**
- 4 NARODOWE CENTRUM BADAN JADROWYCH **PL**
- 5 COMMISSARIAT A L ENERGIE ATOMIQUE ET AUX ENERGIES **FR**
- 6 CENTRUM VYZKUMU REZ SRO **CZ**
- 7 KARLSRUHER INSTITUT FUER TECHNOLOGIE **DE**
- 8 FRAMATOME **FR** Partner
- 9 CESKE VYSOKE UCENI TECHNICKE V PRAZE **CZ**
- 10 EVALION SRO **CZ**
- 11 NSC "KHARKOV INSTITUTE OF PHYSICS AND TECHNOLOGY **UA**
- 12 BUDAPESTI MUSZAKI ES GAZDASAGTUDOMANYI EGYETE **HU**
- 13 SLOVENSKA TECHNICKA UNIVERZITA V BRATISLAVE **SK**
- 14 ZAPADOCESKA UNIVERZITA V PLZNI **CZ**
- 15 STATE OFFICE FOR NUCLEAR SAFETY **CZ**
- 16 HELMHOLTZ-ZENTRUM DRESDEN-ROSSENDORF EV **DE**
- 17 THE CHANCELLOR MASTERS AND SCHOLARS OF THE UNIV **UK**
- 18 THE UNIVERSITY OF SHEFFIELD **UK**

Kharkiv team:

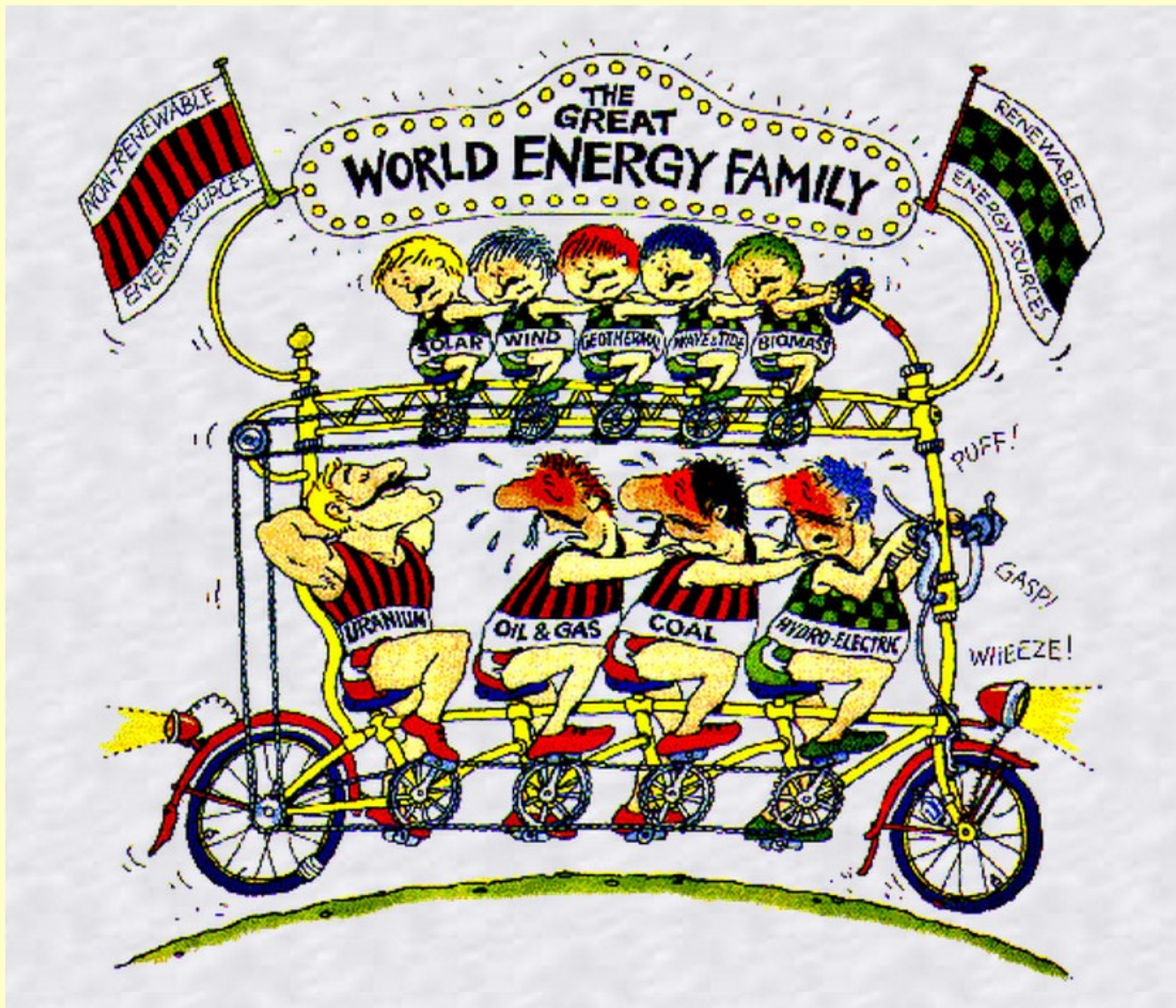
Sergii Fomin - PI

Maksym Malovytsia

Yurii Melnik

Vladimir Pilipenko

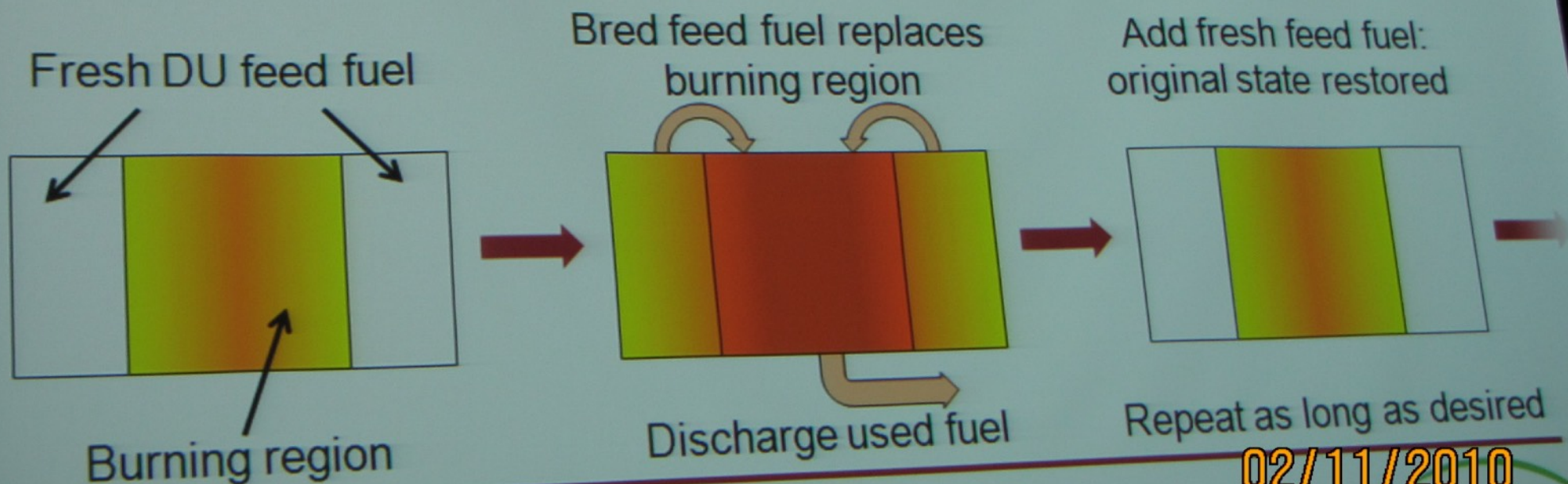
Nikolai Shul'ga



Thank you for attention !

Traveling Wave Reactor Physics

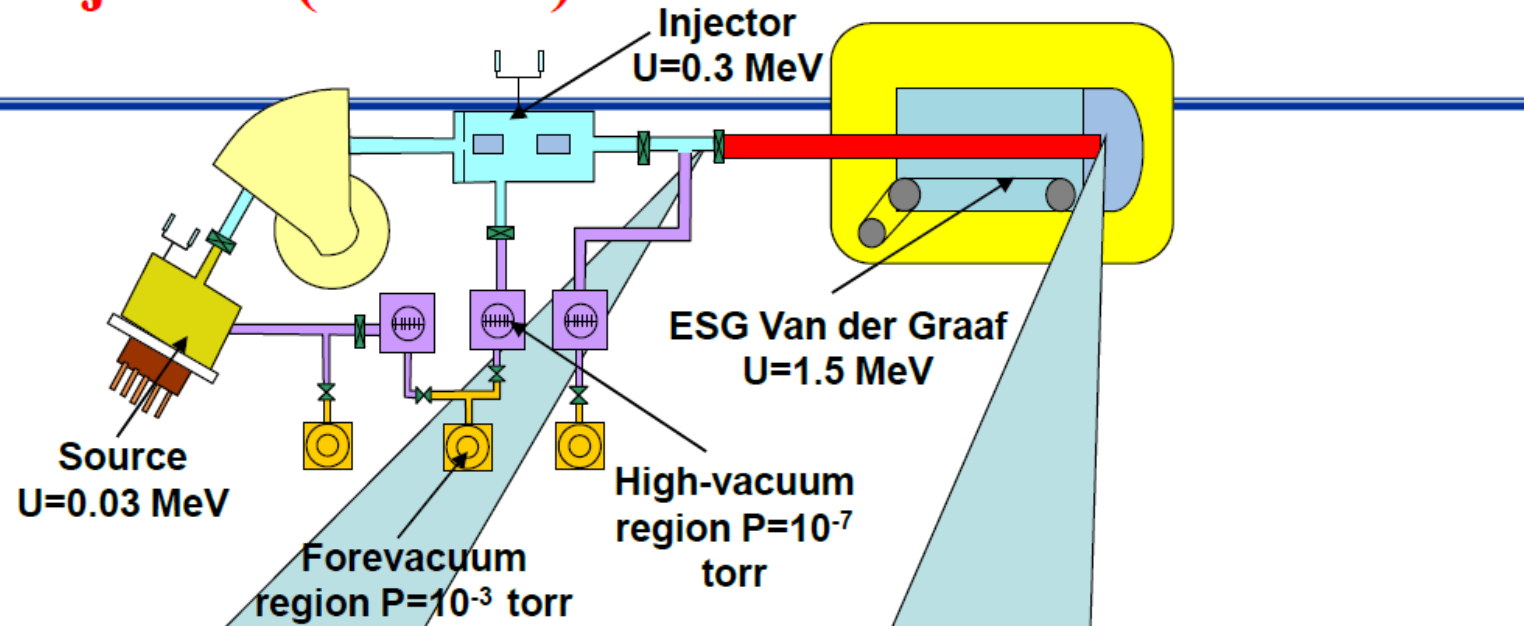
- A *breed-and-burn* reactor:
 - 1. First breed fissile Pu-239 in U-238 fuel, using leakage flux from burning region
 - 2. Newly created fuel can directly replace discharged fuel in burning region and sustain criticality
- **Schematic illustration of a two-zone TWR:**



02/11/2010

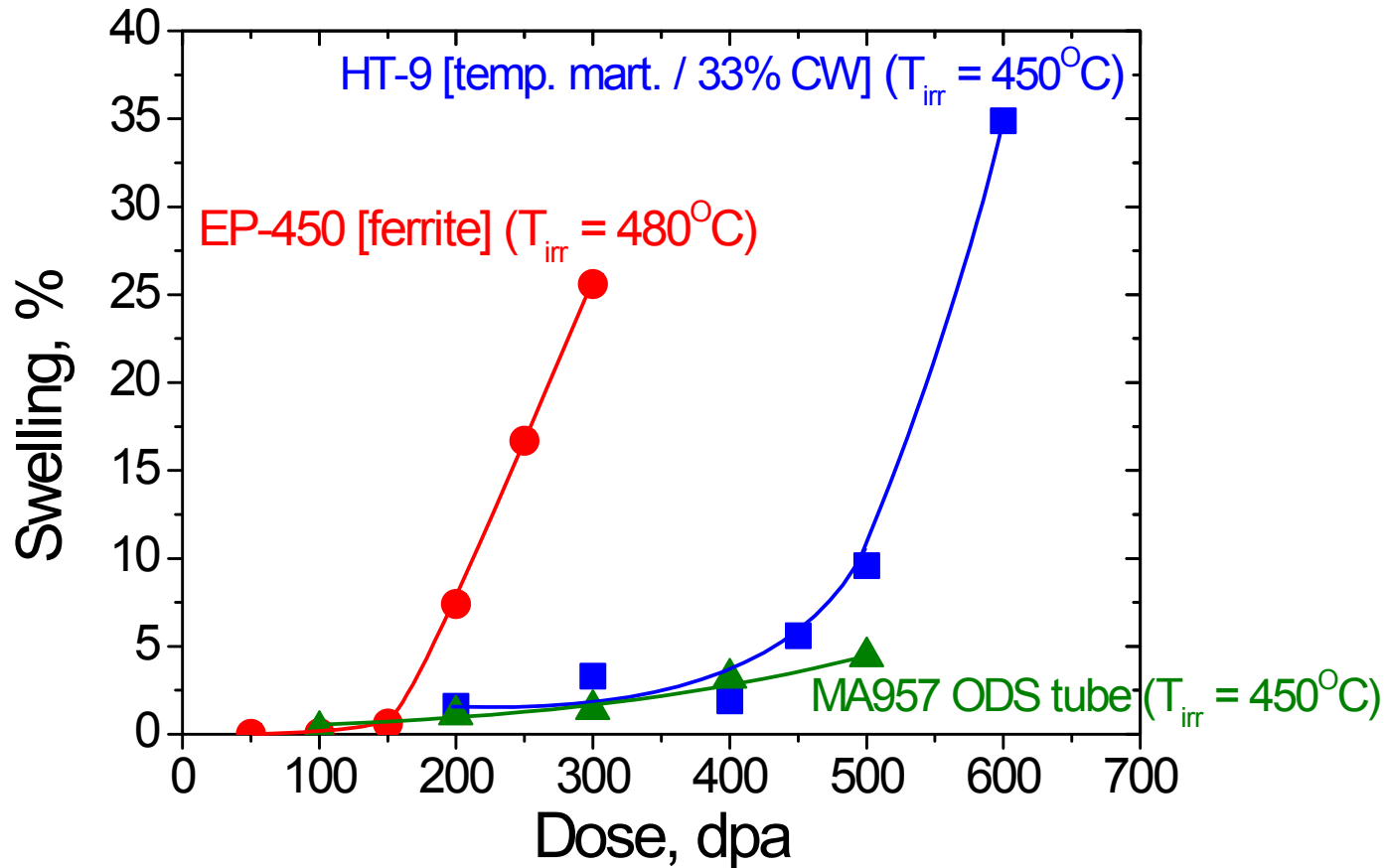


Electrostatic Accelerator with External Injector (ESUVI) at KIPT





Dose dependence of swelling of three ferritic-martensitic steels



Denuded zone effect in very narrow grains depresses the overall swelling somewhat. (ODS - Oxide dispersion-strengthened)